

Exploring Differential Effects of Mathematics Courses on Mathematics Achievement

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Using data from the Longitudinal Study of Mathematics Participation (N = 1,518 students from 34 schools), we investigated the effects of pure and applied mathematics courses on mathematics achievement, controlling for prior mathematics achievement. Results of multilevel modelling showed that the effects of pure mathematics were significant after adjusting for traditionally important student-level predictors of achievement and for school effects. The effects of mathematics courses varied significantly across schools. Students taking pure mathematics achieved higher in smaller schools, particularly schools with higher teacher commitment. Students taking applied mathematics achieved higher if they attended smaller schools.

Keywords: mathematics achievement; mathematics courses; multilevel modeling, Alberta education

À l'aide de données tirées de la *Longitudinal Study of Mathematics Participation* (N = 1518 élèves dans 34 écoles albertaines), les auteurs ont étudié les effets des cours de mathématiques pures et appliquées sur les compétences en mathématiques en tenant compte des compétences antérieures. Les résultats de la modélisation multiveau, une forme des modèles statistiques hiérarchiques linéaires, indiquent que les effets des mathématiques pures sont significatifs une fois pris en compte les prédicteurs de réussite les plus fréquents ainsi que les effets des écoles. Les effets des cours de mathématiques variaient d'une école à l'autre. Les élèves en mathématiques pures ont obtenu de meilleurs résultats dans les écoles de plus petite taille, surtout dans celles où les enseignants s'impliquent davantage. Les élèves en mathématiques appliquées ont eux-aussi obtenu de meilleurs résultats dans les écoles de plus petite taille.

Mots clés : réussite en mathématiques, cours de mathématiques, modélisation multiniveau, éducation en Alberta.

Many researchers have sought important predictors of mathematics achievement, with individual differences being the traditional focus. Among student characteristics influencing mathematics achievement, researchers have paid attention to gender differences (e.g., Ma & Klinger, 2000; Randhawa, 1991), socioeconomic differences (e.g., Crane, 1996; Ma & Klinger, 2000), racial-ethnic differences (e.g., Ma & Klinger, 2000), and differences associated with immigrant status (e.g., Ma & Klinger, 2000; Wang & Goldschmidt, 1999). Meanwhile, researchers have reached consensus that schools are differentially effective in teaching mathematics, as evidenced in the school effectiveness movement (see Teddlie & Reynolds, 2000). Among school characteristics influencing mathematics achievement, researchers have attended to school context and climate (e.g., Lee, 2000; Ma & Klinger, 2000) and school policies and practices (e.g., Lee, Croninger, & Smith, 1997; Luyten, 1994).

Researchers have also investigated the effects of mathematics coursework on mathematics achievement, examining the impact of the number (amount) and the type (content) of mathematics courses. Sebring (1987) noted that the quantity of mathematics coursework had an effect on mathematics achievement. Gamoran (1987) reported that students performed better on standardized mathematics achievement tests if they took more mathematics courses, and mathematics coursework particularly differentiates mathematics achievement between White and Black students. Grouping students based on gender (male and female) and race (white and non-white), Jones (1987) claimed that across all four groups, students with more mathematics courses had higher mathematics achievement.

Overall, there has been a push to raise the academic standards in mathematics by increasing the number of mathematics courses students must complete to graduate from high school. National studies in the United States showed that, after this requirement was implemented, the percentage of students taking pre-calculus changed from 15 per cent to 17 per cent, and for calculus the change was from 8 per cent to 11 per cent (see Hoffer, 1997). However, many schools have responded to this requirement by offering more low-level mathematics courses. Hoffer (1997) attributed this situation to the fact that "the school lacks the instructional resources [e.g., hiring additional teachers] to implement the

policy properly” and “students are unwilling or unable to handle more difficult mathematics” (p. 587). According to Hoffer (1997), this “watered-down” requirement explains why the move to augment mathematics standards by increasing the number of mathematics courses students must complete has not increased mathematics achievement. He suggested that it is important to move “beyond simple course-count requirements to an emphasis on specific curriculum content and actual learning outcomes” (p. 48). Perhaps it is the type or content of mathematics courses that produces differences in mathematics achievement.

Some studies have considered the differential effects of mathematics courses on students’ subsequent mathematics achievement. In general, the type of mathematics courses students complete in high school is strongly associated with their mathematics achievement. For example, analytic geometry and calculus are related to students’ academic achievement (Stribling, 1990). Smith (1996) found that students who took formal algebra courses early (in grade 8) outperformed students who did not on standardized mathematics achievement tests in both grades 10 and 12. Ma (2000) reported that every advanced mathematics course (algebra II, trigonometry, analytic geometry, and calculus) has an effect on mathematics achievement, even after accounting for traditionally important predictors such as socio-economic status (SES) and prior mathematics achievement. He concluded that “the effect of mathematics coursework was over and above the effects of academic background and individual characteristics” (p. 26). Consistently, these researchers described the content effects of mathematics courses as either strong or substantial.

These studies primarily examined the effects of mathematics coursework on mathematics achievement from the student perspective. However, school effects may be as important as individual differences. School characteristics may mediate the relationship between mathematics coursework and mathematics achievement. School policies and practices on course offering and taking are certainly important to this relationship. As Smith (1996) asserted, the mathematics courses schools offer can influence students’ academic achievement and career aspiration. Gamoran (1987) believed that “curriculum differentiation

(tracking) may create stratified learning opportunities within schools" (p. 135). This is particularly detrimental to the academic success of low-achieving students because "they are typically tracked into low-level, dead-end mathematics classes" (Gamoran, Porter, Smithson, & White, 1997, p. 325). Lee, Croninger, and Smith (1997) support this conclusion:

Students are advantaged by attending schools where they and their classmates take more academic mathematics courses, in schools where more students pursue their studies within a college-preparatory program, and in schools whose mathematics curricula consist of higher proportions of academic courses. (p. 112)

What is lacking in the research literature is how school context and climate mediate the relationship between mathematics coursework and mathematics achievement. Our study adds empirical evidence from the perspective of school context and climate to the relatively thin research literature on the relationship between the content of mathematics coursework and the level of mathematics achievement. It is a direct response to the shift in research focus from the number to the type of mathematics courses, which has not yet stimulated sufficient studies that investigate the content of mathematics coursework as it relates to the level of mathematics achievement.

In particular, almost all research on mathematics coursework is conducted outside Canada. Several provinces have adopted the policy and course offering of the pure/applied/essentials streams. Whether intentional or not, such a new policy focuses on content rather than number of mathematics courses. This offers Canadian researchers an opportunity to be a part of the ongoing debate on content versus number in mathematics coursework. Central to this shift is the question: Can this new coursework policy improve mathematics achievement of Canadian students? Focusing on students at the beginning of high school, we specified three research questions to address (a) whether taking different mathematics courses made a significant difference in mathematics achievement, (b) whether these coursework effects varied significantly across schools, and (c) whether school characteristics accounted for the variation if these coursework effects varied across schools.

METHOD

Subjects

Data were drawn from the Longitudinal Study of Mathematics Participation (LSMP), a large-scale research project at the University of Alberta. Tenth-grade students were followed until their graduation year to identify student, teacher, and school characteristics that contributed to students' decision to participate in mathematics courses. The school was the sampling unit. Rogers and Wilson (1993), sampling schools from the central part of the province of Alberta, demonstrated that analytic results based on such a sampling strategy are representative of the entire province. For this reason, the LSMP adopted the same strategy, concentrating on the central part of the province. Schools were approached through school districts, with offers of financial compensation for interruption of regular instruction. As a result, 34 schools with grades 10 to 12 from 11 school districts participated in the LSMP.

Within each school, all students in grade 10 were invited to participate with parental consent. School contact persons (mathematics teachers in most cases) worked with members of the research team to distribute and collect parental consent letters. To encourage participation, free lunch was offered to students. Student participation rate varied from 40 per cent to 100 per cent across schools. School contact persons were asked to evaluate how representative students with parental consent were of all tenth graders in their schools and were particularly instructed to take note of patterns of non-participants (e.g., low SES students or students with learning difficulties in mathematics). Even in schools with relatively low participation rates, school contact persons were quite confident that non-participation did not indicate any evident concentration of students with common characteristics such as low SES or learning difficulties. The base-year data of the LSMP contained 1,518 students, representing 45 students per school. In addition, 155 mathematics teachers and 34 principals of participating students also participated.

Materials and Procedures

Students completed a standardized mathematics achievement test. One questionnaire was administered to students, teachers, and principals to collect information on student and school characteristics expected to influence mathematics coursework. Mathematics achievement was the dependent variable, measured with a mathematics subtest of the *Canadian Achievement Tests* (2nd ed.) (CAT/2) (Canadian Test Centre, 1992). The test is age-appropriate to sampled students and assessed their ability to understand mathematical concepts and apply this understanding to solve mathematical problems. CAT/2 is essentially a skill (or ability) test, not influenced by curricular content (Canadian Test Centre, personal communication, September, 1999). Therefore, testing results were not biased against students in any course.

Student-level characteristics included continuous variables of age, mother SES, father SES, number of parents, and number of siblings as well as dichotomous variables of gender, immigrant student, Aboriginal student, or minority student. All student-level variables were self explanatory in meaning, coming from single items on the student questionnaire, except SES. The conventional measure of SES (for example, Duncan's socioeconomic index) is a standardized composite variable of parental education, parental occupation, and household income. Because the research team was unable to collect information on household income, parental education and occupation were combined to create a standardized composite variable of SES following statistical procedures outlined in Mosteller and Tukey (1977).

Another important variable was prior mathematics achievement. Because the research team was unable to access student records, students were asked to provide final marks (in percentage points as commonly used in participating schools) in mathematics in the previous year. We used prior mathematics achievement to avoid a potential temporal problem that the superior performance of students in advanced mathematics courses was a result of their superior prior performance (which may actually lead them to attend advanced mathematics courses). With prior mathematics achievement functioning as a covariate or control variable, differences in mathematics achievement were more likely attributed to course attendance than prior performance. For this

reason, prior mathematics achievement was present in all statistical analyses involving mathematics coursework so that the effects of mathematics courses on mathematics achievement could be adjusted for prior mathematics ability.

Mathematics coursework was a categorical variable made up of three categories: pure mathematics, applied mathematics, and low-level preparatory mathematics courses that prepare students to enroll in a formal mathematics course (i.e., pure or applied mathematics). The provincial curriculum guides describe the rationale and content of pure and applied mathematics.

Pure mathematics emphasizes mathematical theory and the testing of mathematical hypotheses. The pure mathematics approach, which is often deductive and symbolic, endeavours to show that concepts are valid all the time, or valid within a well-defined set of restrictions. Real-life problems are then presented in order for students to apply previously learned mathematical concepts and procedures. (Alberta Learning, 2002b, p. 1)

Applied mathematics gives students a clearer picture of why they are learning mathematics and motivates them in learning. Students experience mathematics as being dynamic and useful in their careers and everyday life. The approach used in applied mathematics is primarily data driven, using numerical and geometrical problem-solving techniques. As a way of increasing relevance, students collect data in experiments and activities and develop mathematics concepts from analyses of the data. (Alberta Learning, 2002a, p. 1)

A table outlining the expected general outcomes for each of the three mathematics courses is available from the authors. We created two dichotomous variables to represent pure and applied mathematics courses, with low-level preparatory mathematics courses as the baseline effect. At the student level, continuous variables (including mathematics achievement) were standardized to have a mean of zero and a standard deviation of one, and dichotomous variables were centred around their grand means.

School-level characteristics included variables measuring school context and climate. School context included such variables as school size, school location, school mean SES, and material resources for mathematics. We have provided a description of school climate

measures (composite variables made from scales containing multiple items) in Appendix A. School-level variables were centred around their grand means.

Some school-level variables, such as school SES and parental involvement, seem equally logical as student-level variables. Studies of school effects focus on how these variables define school context and climate (see Teddlie & Reynolds, 2000). For example, school SES measures school socioeconomic composition, and parental involvement is one of the most important measures of school climate (see Ma, 1999). In other words, these variables represent social and educational policy issues more than issues of individual differences. This treatment generates knowledge about how to help students in socially disadvantaged schools and how parental involvement can create a school-wide impact on academic achievement of all students.

Data Analysis

Because our data were hierarchically structured with students nested within schools, we used multilevel modelling techniques (Raudenbush & Bryk, 2002) to simultaneously determine which student (level one) and school (level two) characteristics significantly predicted mathematics achievement (see Kreft & De Leeuw, 1998).

We tested four multilevel models. The first model, containing student-level variables with control for prior mathematics achievement, informed us of the effects of student background variables on mathematics achievement (after adjustment for prior mathematics achievement).

$$achievement_{ij} = \beta_{0j} + \beta_{1j}prior_{ij} + \sum \beta_{pj}X_{pij} + \epsilon_{ij}$$

$$\beta_{0j} = \gamma_{00} + \mu_{0j}$$

The first equation is the student model that specifies that mathematics achievement of student i in school j is influenced by prior mathematics achievement and background variables (X_p) of the student (an error term is also present to capture all possible errors due to uncontrolled student characteristics). The second equation is the school model where the

intercept (representing school average mathematics achievement) is allowed to vary among schools (an error term is also present to capture all possible errors due to uncontrolled school characteristics). No school-level variables were used at this stage to model this variation.

We then added mathematics coursework indicators to statistically significant student-level variables and prior mathematics achievement in the second model that informed us of the effects of mathematics courses on mathematics achievement in the presence of student background variables with adjustment for prior mathematics achievement (i.e., after controlling for student background variables and prior mathematics achievement).

$$achievement_{ij} = \beta_{0j} + \beta_{1j}pure_{ij} + \beta_{2j}applied_{ij} + \beta_{3j}prior_{ij} + \sum \beta_{pj}X_{pji} + \epsilon_{ij}$$

$$\beta_{0j} = \gamma_{00} + \mu_{0j}$$

$$\beta_{1j} = \gamma_{10} + \mu_{1j}$$

$$\beta_{2j} = \gamma_{20} + \mu_{2j}$$

The first equation specifies that mathematics achievement of student i in school j is influenced by whether the student enrolled in pure or applied mathematics as well as prior mathematics achievement and background variables of the student. The group of equations that follows the first equation is the school model where the intercept and coefficients associated with coursework indicators (representing coursework effects) are allowed to vary among schools (error terms are also present to capture all possible errors due to uncontrolled school characteristics). No school-level variables were used at this stage to model these variations.

In the next model, we added school-level variables to examine whether the presence of school effects (on mathematics achievement) could alter the effects of mathematics coursework indicators on mathematics achievement,

$$achievement_{ij} = \beta_{0j} + \beta_{1j}pure_{ij} + \beta_{2j}applied_{ij} + \beta_{3j}prior_{ij} + \sum \beta_{pj}X_{pji} + \epsilon_{ij}$$

$$\beta_{0j} = \gamma_{00} + \sum \gamma_{q0j} W_{qj} + \mu_{0j}$$

$$\beta_{1j} = \gamma_{10} + \mu_{1j}$$

$$\beta_{2j} = \gamma_{20} + \mu_{2j}$$

where W_j represent school-level variables used to model variation in school average mathematics achievement.

In the final multilevel model, we used school-level variables to model variation in coursework effects associated with pure and applied mathematics.

$$achievement_{ij} = \beta_j + \beta_{1j} pure_{ij} + \beta_{2j} applied_{ij} + \beta_{3j} prior_g + \sum \beta_{pj} X_{pij} + \epsilon_{ij}$$

$$\beta_{0j} = \gamma_{00} + \mu_{0j}$$

$$\beta_{1j} = \gamma_{10} + \sum \gamma_{q0j} W_{qj} + \mu_{1j}$$

$$\beta_{2j} = \gamma_{20} + \sum \gamma_{q0j} W_{qj} + \mu_{2j}$$

We took this systematic approach to test the impact of mathematics coursework on mathematics achievement.

RESULTS

Table 1 shows descriptive statistics for the independent variables, together with coding information for all dichotomous variables. A mean for a dichotomous variable represents the proportion of the group coded as one. These descriptive statistics were in their original scales, except for mother SES and father SES, which were standardized variables with a mean of zero and a standard deviation of one. Before multilevel analysis, we examined correlations among both student-level and school-level variables and found no high correlations at either level (the highest correlation was 0.24 at the student level and 0.67 at the school level).

TABLE 1
Descriptive Statistics of Student and School Variables

Variable	Mean	SD
Student variables		
Grade 10 pure mathematics (taking = 1, not taking = 0)	0.62	0.49
Grade 10 applied mathematics (taking = 1, not taking = 0)	0.11	0.32
Gender (female = 1, male = 0)	0.50	0.50
Age	15.54	0.62
Number of parents	1.83	0.39
Number of siblings	2.24	1.65
Immigrant student (immigrant = 1, non-immigrant = 0)	0.05	0.22
Aboriginal student (aboriginal = 1, non-aboriginal = 0)	0.06	0.24
Minority student (minority = 1, non-minority = 0)	0.08	0.28
Prior mathematics achievement	73.75	13.64
School variables		
School size	612.76	362.77
School location (urban = 1, rural = 0)	0.62	0.49
Material resources for mathematics	2.47	0.55
Academic expectation	4.21	0.44
Disciplinary climate	3.00	0.38
Parental involvement	2.23	0.62
Principal instructional leadership	3.97	0.61
Teacher commitment	4.21	0.58
Teacher morale	4.33	0.49
Teacher job satisfaction	2.13	0.48
Teacher control	4.17	0.43
Teacher influence	2.72	0.48

Note: In addition, father and mother socioeconomic status (SES) are standardized variables (with a mean of zero and a standard deviation of one) at the student level, and school mean father and mother SES are standardized variables at the school level.

Table 2 presents the proportion of variance in mathematics achievement explained by three multilevel models: (a) taking only student background variables into account, (b) taking statistically

significant student background variables (gender, age, immigrant student and Aboriginal student) and mathematics courses into account, and (c) taking statistically significant student background variables, mathematics courses, and school-mean mother SES (the only statistically significant school-level variable) into account. All three models controlled for prior mathematics achievement at the student level. To save space, this control will no longer be emphasized even though prior mathematics achievement was present in all statistical analyses involving mathematics courses.

TABLE 2

Comparison of Proportion of Variance in Mathematics Achievement Explained by Various Multilevel Models

	Student variables alone	Student variables with coursework	Student and school variables with coursework
Between students	0.28	0.31	0.31
Between schools	0.33	0.38	0.48

Note: Student variables include gender, age, immigrant student, and aboriginal student with control for prior mathematics achievement. School variables include school mean mother SES. These variables are statistically significant at the alpha level of 0.01.

The model with student background as student-level variables accounted for 28 per cent of the variance in mathematics achievement between students and 33 per cent between schools. In the next model, mathematics courses were added to statistically significant student background variables. Mathematics courses added 3 per cent of the explained variance in mathematics achievement between students and 5 per cent between schools. Therefore, mathematics coursework demonstrated unique effects on mathematics achievement even after student background variables were controlled.

The last model contained mathematics courses as well as statistically significant student-level and school-level variables. We added school-

level variables to model the intercept (school average mathematics achievement) to examine how school effects on mathematics achievement could change the effects of mathematics coursework on mathematics achievement. This model accounted for 31 per cent of the variance in mathematics achievement between students and 48 per cent between schools. We found no improvement in the explained variance at the student level. The improvement in the explained variance between schools was obviously a result of the direct modelling of school average mathematics achievement by school-level variables. Because no change occurred in the explained variance between students, we concluded that the importance of mathematics coursework to mathematics achievement as reported above remained unchanged even after adjustment for school effects on mathematics achievement. Overall, mathematics coursework shows important unique effects on mathematics achievement even in the presence of individual differences in and school effects on mathematics achievement.

Table 3 corresponds to the second and fourth multilevel models, showing the fixed and random effects of mathematics coursework on mathematics achievement. A fixed effect (on the dependent variable) is expressed as a proportion of a standard deviation associated with one standard deviation increase in an independent variable. Even in the presence of student background variables, students who attended pure mathematics achieved 0.45 *SD* (i.e., 45% of a standard deviation) higher in mathematics achievement than students who attended low-level preparatory mathematics courses. However, students who attended applied mathematics did not perform any better than students who attended low-level preparatory mathematics courses. Such a pattern remained unchanged after adjustment for both student-level and school-level variables (with the effect of pure mathematics being 0.48 *SD* and no effect of applied mathematics).

The random effects of mathematics coursework indicate the extent to which the effects of mathematics courses varied across schools. The model adjusted for student-level variables showed that the effects of pure mathematics varied significantly across schools with one standard deviation as 0.28 *SD* in effect (the square root of the variance 0.08). Some schools were doing better than others in using pure mathematics to

promote mathematics achievement. Although the average effect of applied mathematics was not significant, the effects of applied mathematics did vary across schools with one standard deviation as 0.33 *SD* in effect (the square root of the variance 0.11). Some schools were more successful than others in using applied mathematics to promote mathematics achievement. Overall, mathematics coursework appears to be important to mathematics achievement (e.g., the same course was related to different levels of achievement across schools). This conclusion is consistent with that obtained earlier through the comparison of variance explained by various models.

TABLE 3

Effects of Mathematics Courses on Mathematics Achievement and Variation in Effects Across Schools, with Control for Prior Mathematics Achievement

	Adjusted for student variables		Adjusted for student and school variables	
	<i>Effect</i>	<i>SE</i>	<i>Effect</i>	<i>SE</i>
<i>Average effects of coursework</i>				
Grade 10 pure mathematics	0.45***	(0.08)	0.48***	(0.07)
Grade 10 applied mathematics	0.07	(0.11)	0.07	(0.12)
<i>Variation in effects across schools</i>				
Grade 10 pure mathematics	0.08	39.44**	0.03	28.21
Grade 10 applied mathematics	0.11	37.87**	0.12	34.95**

Note: ** $p < 0.01$. *** $p < 0.001$. Both models (adjusted for student variables and adjusted for student and school variables) control for prior mathematics achievement.

Once school-level variables were added (see the fourth multilevel model), schools were no longer variable in the effects of pure mathematics (variance became non-significant). Schools were still variable in the effects of applied mathematics with one standard deviation as 0.35 *SD* in effect. Therefore, we identified responsible school-level variables in the case of pure mathematics in that school-level variables successfully accounted for all the significant variation across schools in the effects of pure mathematics on mathematics achievement.

Yet, school-level variables could not account for the significant variation across schools in the effects of applied mathematics on mathematics achievement.

Additional statistical results from the last multilevel model, presented in Table 4, illustrate those school-level variables responsible for significant variations across schools in the effects of mathematics coursework on mathematics achievement. Four statistically significant variables occurred at the student level (gender, age, immigrant student, and Aboriginal student). These variables were not interpreted because they were used as control variables (together with prior mathematics achievement) to show that after controlling for these significant student background variables, students who attended pure mathematics courses still achieved 0.48 *SD* higher in mathematics than students who attended low-level preparatory mathematics courses. In contrast, after adjustment for student background variables, students who attended applied mathematics courses did not perform any better than students who attended low-level preparatory mathematics courses.

Two statistically significant school-level variables were responsible for the significant variation across schools in the effects of pure mathematics on mathematics achievement. First, the effects of pure mathematics were smaller in larger schools. Consider two schools with a difference of 100 students in enrolment. Students taking pure mathematics in the smaller school would achieve 0.05 *SD* higher in mathematics than students taking pure mathematics in the larger school. Second, the effects of pure mathematics were larger in schools where teacher commitment was higher. Teacher commitment was measured on a scale from 1 to 5 (see Appendix A). Consider two schools with a difference of one score point in teacher commitment. Students taking pure mathematics in the school with better teacher commitment would achieve 0.22 *SD* higher in mathematics than students taking pure mathematics in the school with poorer teacher commitment. Therefore, small schools and schools with high teacher commitment were more successful in using pure mathematics to promote mathematics achievement. The effects of teacher commitment were more than four times as strong as the effects of school size, indicating that teacher commitment was a much more important school-level variable than

Table 4

Results of School Variables Explaining Variation in Effects of Mathematics Courses on Mathematics Achievement Across Schools, with Control for Prior Mathematics Achievement and Adjustment for Student Variables

Effects of mathematics courses on mathematics achievement		
Grade 10 pure mathematics	0.48***	(0.09)
Grade 10 applied mathematics	0.07	(0.11)
Effects of student variables on mathematics achievement		
Gender (female)	-0.14***	(0.04)
Age	-0.14***	(0.04)
Immigrant student	-0.20*	(0.08)
Aboriginal student	-0.31***	(0.09)
Prior mathematics achievement	0.37***	(0.05)
Effects of school variables on the slope of Grade 10 pure mathematics		
School size	-0.05***	(0.01)
Teacher commitment	0.22**	(0.07)
Effects of school variables on the slope of Grade 10 applied mathematics		
School size	-0.05**	(0.02)

Note: * $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$. School size is used as the number of units with 100 students as one unit. School size unit is increased from 1 to 100 to magnify the effect of school size.

school size in terms of explaining the effects of pure mathematics on mathematics achievement.

The effects of applied mathematics were also smaller in larger schools. Consider again two schools with a difference of 100 students in enrolment. Students taking applied mathematics in the smaller school would achieve 0.05 *SD* higher in mathematics than students taking

applied mathematics in the larger school. Therefore, small schools were more successful in using applied mathematics to promote mathematics achievement. The effects of school size were identical between the pure and applied mathematics cases. Small schools were capable of using both pure and applied mathematics to promote similar improvement in mathematics achievement.

DISCUSSION

Summary of Principal Findings

Mathematics coursework had critical effects on mathematics achievement. The average effect of pure mathematics was significant even after controlling for prior mathematics achievement and traditionally important predictors of mathematics achievement at the student level as well as school effects on mathematics achievement. The effects of pure mathematics varied significantly across schools. School size and teacher commitment were responsible for this variation. Students taking pure mathematics in smaller schools or in schools with better teacher commitment achieved higher in mathematics than students taking pure mathematics in larger schools or schools with less teacher commitment.

Specifically, a difference of 100 students in enrolment was associated with a difference in mathematics achievement of 5 per cent of a standard deviation. A quarter of a standard deviation often indicates a difference that is substantial enough to warrant practical implications. To reach that level, a reduction in school size between 400 and 500 students is required, which may not be possible practically. In this sense, the effect of school size can be labelled as small. In contrast, a difference of one score point on a five-point scale of teacher commitment was associated with a difference in mathematics achievement of 22 per cent of a standard deviation (neighbouring a quarter of a standard deviation), indicating that improving teacher commitment is a very achievable strategy to increase the effectiveness of pure mathematics (and substantial improvement in mathematics achievement can be anticipated).

The average effect of applied mathematics was not significant on mathematics achievement once we controlled for prior mathematics

achievement and significant student background variables. The effects of applied mathematics did vary significantly across schools. School size accounted only for a portion of this variation leaving a significant amount of the variance unexplained. Students taking applied mathematics in smaller schools achieved higher in mathematics than did students taking applied mathematics in larger schools. A difference of 100 students in enrolment was associated with a difference in mathematics achievement of 5 per cent of a standard deviation. Similar to the case of pure mathematics, this effect of school size can be considered small.

Content and Amount of Mathematics Coursework

Our results support the argument that mathematics achievement may be greatly influenced by the content rather than the amount of mathematics coursework (e.g., Hoffer, 1997). Although we did not directly compare the effects on mathematics achievement between the content and amount of mathematics coursework, we did provide empirical evidence on the critical role that the content of mathematics coursework plays in promoting mathematics achievement. Students completing pure mathematics demonstrated substantially (almost half of a standard deviation) higher mathematics achievement than students completing preparatory mathematics courses, but in contrast, students completing applied mathematics did not show any better mathematics achievement than students completing preparatory mathematics courses. The significant effects of pure mathematics stand out evidently given that we controlled for prior mathematics achievement when modelling the impact of mathematics coursework. We conclude that the content of mathematics courses matters to mathematics achievement.

The pure mathematics course “emphasizes mathematical theory and the testing of mathematical hypotheses” (Alberta Learning, 2002b, p. 1). This course encourages students to approach mathematical concepts and problems in a deductive and symbolic manner. The applied mathematics course, on the other hand, emphasizes a more practical approach to mathematical concepts and problems (Alberta Learning, 2002a). This course emphasizes how mathematics can be useful in everyday life and in future careers. It appears that an emphasis on the theoretical aspect of

mathematics (as in pure mathematics), rather than the practical aspect of mathematics (as in applied mathematics), relates more strongly with student mathematics achievement even after taking into account prior mathematics ability. Research evidence exists that supports this distinction (between theoretical and practical approaches). For example, the Third International Mathematics and Science Study (TIMSS) 1999 video study showed that one of the most distinguishable instructional characteristics of the high-achieving countries (Australia, Czech Republic, Hong Kong, Japan, Netherlands, and Switzerland) is that most instructions are presented in mathematical symbols rather than everyday-life contexts (see Hiebert et al., 2003).

More important than offering our empirical support, we have demonstrated that the effects of pure mathematics on mathematics achievement were actually quite stable or robust (even after taking into account important student-level and school-level variables). Particularly, the effects of pure mathematics on mathematics achievement are over and above the effects of student and school characteristics. This conclusion has an implication for educational policymakers. If the content of mathematics courses in high school is so important to student mathematics achievement, then students should be required to increase the level of mathematics courses they take for graduation (see also Sebring, 1987).

School Effects

Based on our finding that the effects of mathematics coursework (both pure and applied mathematics) varied significantly across schools, we suggest that some schools were more successful in using mathematics coursework to promote mathematics achievement than other schools (even though the average effect of applied mathematics was not significant). This conclusion supports the important role of schools in shaping the relationship between mathematics coursework and mathematics achievement. Our analysis extends the argument of Gamoran et al. (1997) and Lee et al. (1997) that schools can use course offering to influence student mathematics achievement. We have shown that even when schools offer the same mathematics courses, they can still be differentially effective in using these mathematics courses to influence

mathematics achievement.

An additional important finding of the current study is that we identified effective schools in using mathematics coursework to influence mathematics achievement. Students who take the pure mathematics course are more likely to achieve higher in mathematics achievement if they attend smaller schools, particularly if they attend schools with higher teacher commitment. Students who take the applied mathematics course can also be expected to achieve higher in mathematics if they attend smaller schools. Small schools with highly committed teachers are in an advantageous position to help students succeed in learning pure mathematics. We suggest that promoting teacher commitment to education is critical for students enrolled in pure mathematics to succeed. School size appears as the only significant school-level variable across both pure and applied mathematics. Although it is not practical to advocate downsizing schools (a reduction between 400 and 500 students in our case), we believe that some lessons about small schools benefit all schools. We want to emphasize two important characteristics regarding personal interactions in small schools which may be offered as a possible cause for the greater effects associated with smaller schools. Often, smaller schools can afford more personal interactions that can also spread over a longer period of time than larger schools. The importance of homerooms may need to be realized in large schools. Large schools may centre their teaching and learning activities more around homerooms so that students can have increased and prolonged personal interactions with both teachers and peers.

Although we included ten school-level variables that portray the climate or learning environment of a school, we detected only one variable statistically significant in predicting the effects of mathematics coursework on mathematics achievement. In a meta-analysis of school effectiveness, Bosker and Witziers (1996) reported that school effects typically account for 10 per cent of the variation in student academic achievement. They highlighted that school effects are greater for mathematics than for language and concluded that school factors do matter to mathematics achievement. Several specific studies related to the effects of school climate occur in the research literature. For example,

academic expectation (Zigarelli, 1996), disciplinary climate (Ma & Klinger, 2000), parental involvement (Thacker, 2000), and teacher commitment (Reyes & Fuller, 1995) have been identified as significant school-level predictors of mathematics achievement.

Based on these studies, we expected that some school climate characteristics would significantly strengthen the effects of mathematics coursework on mathematics achievement. The fact that only teacher commitment showed such a function may be a result of schools being fairly similar in school climate. The small standard deviations among schools in school climate variables indeed indicate such a possibility (Table 1). Alternatively, we question whether school climate was measured adequately in the LSMP. Most scales measuring school climate have a small number of items, which may be offered as another possible explanation for the lack of significant school climate variables.

Limitations and Recommendations

The limitation of the current study resides with sampling and measurement. Students and schools participating in the LSMP constitute a volunteering rather than random sample, and we do not have means to compare this sample with the general population of students and schools in the province of Alberta. In addition, the return rate in some schools was low. Both facts speak to the limitation in terms of data collected in the LSMP. As mentioned earlier, there is the limitation in terms of measurement used in the LSMP. Further investigations are encouraged to pursue more representative samples of students and schools and develop more sensitive scales for measuring important constructs such as school climate.

Another limitation is related to the design of the analysis. Students were not randomly assigned to mathematics courses (i.e., pure, applied, and preparatory mathematics). We used prior mathematics achievement to alleviate confounding effects coming from prior mathematics ability or curriculum streaming (into those mathematics courses). Although this strategy is statistically adequate, our measure of prior mathematics achievement may not be as perfect as one would expect. Further investigations are encouraged to develop better measures of prior ability or streaming to produce “purer” effects of mathematics coursework.

The analytical design as discussed above also brings up the appropriateness of the term “effect.” Some researchers prefer to reserve this term for a true experimental design involving manipulation of variables. At its very best, the current analysis is a quasi-experimental design in which the term “effect” does not mean that an effect has been caused via manipulation. This fact speaks to two issues. First, we used the term “effect” in a loose manner for convenience without any causal implications (as it has been popularly used in the research literature). Second, we recommend that a true experimental design may need to be pursued. The combination of small-scale true experimental designs and large-scale quasi-experimental designs (like ours) may hold the key to eventually unwrap the complex relationship between mathematics coursework and mathematics achievement.

Finally, from a substantive perspective, we recommend that further investigations examine whether school curricular and instructional characteristics can strengthen the effects of mathematics coursework on mathematics achievement. An alternative explanation for the lack of significant school-level climate variables is that school curricular and instructional characteristics may be more responsible for the effects of mathematics coursework than school climate. After all, mathematics teachers may put different curricular emphases on different topics, and the same enacted content may also be instructed differently. It is, therefore, important to examine the effects of mathematics coursework under different curricular and instructional conditions.

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APPENDIX A: DESCRIPTION OF COMPOSITE VARIABLES AT THE SCHOOL LEVEL

Material Resources for Mathematics

Please indicate the extent to which each of the following school resources meets your instructional needs in mathematics: (a) Curricular resources (e.g., documents, guides, manuals); (b) Computers; (c) Computer software; (d) Calculators; (e) Manipulatives; (f) The library. (1 = does not meet, 2 = partially meets, 3 = adequately meets, 4 = completely meets)

Academic Expectation

(a) I have high expectations for the academic success of my students; (b) I encourage students to do extra work so they can get better grades; (c) I push students to achieve their full academic potential; (d) I believe all students in my class can master the curriculum; (e) I place greater emphasis on developing students' social and personal skills than their academic skills. (1 = strongly disagree, 2 = disagree somewhat, 3 = neutral, 4 = agree somewhat, 5 = strongly agree)

Disciplinary Climate

To what extent is each of the following matters a problem in your class: (a) Student tardiness; (b) Student absenteeism; (c) Cheating on tests; (d) Students disrupting the class; (e) Student apathy; (f) Conflicts among students. (1 = a serious problem, 2 = a moderate problem, 3 = a minor problem, 4 = not a problem)

Parental Involvement

Since the beginning of this school year, approximately how many students' parents or guardians have talked with you concerning their child's: (a) Academic performance; (b) Coursework in mathematics; (c) Behaviour in classroom or school; (d) Social relationships or interactions; (e) Absence from class? (1 = none, 2 = 1-4, 3 = 5-9, 4 = 10-14, 5 = 15-19, 6 = 20+)

Principal Instructional Leadership

(a) My principal has communicated what kind of school he or she wants to the staff; (b) My principal's beliefs about what the main goals of our school should be are the same as mine; (c) My principal lets staff members know what is expected of them; (d) My principal does a poor job of getting resources for this school; (e) My principal enforces school rules for student conduct and backs me up when I need it; (f) My principal talks with me frequently about my instructional practices; (g) My principal provides support and encouragement. (1

= strongly agree, 2 = somewhat agree, 3 = neutral, 4 = somewhat disagree, 5 = strongly disagree)

Teacher Commitment

(a) I am committed to making our school one of the best in the province; (b) If I could start over, I would become a teacher again. (1 = strongly disagree, 2 = disagree somewhat, 3 = neutral, 4 = agree somewhat, 5 = strongly agree)

Teacher Morale

(a) I look forward to each day at school; (b) Teachers at this school have a positive attitude towards students; (c) Teachers at this school work hard to help their students succeed; (d) Our staff room has a friendly atmosphere; (e) Rules for student behaviour are consistently enforced by teachers in this school, even for students who are not in their classes; (f) There is a great deal of cooperative effort among the staff members. (1 = strongly disagree, 2 = disagree somewhat, 3 = neutral, 4 = agree somewhat, 5 = strongly agree)

Teacher Job Satisfaction

(a) I find my professional role satisfying; (b) My success as a teacher is often hindered by having to do administrative tasks; (c) I have to follow rules in this school that conflict with my best professional judgment; (d) Most of my colleagues share my beliefs and values about what the central mission of the school should be; (e) In this school, staff members are recognized for a job well done. (1 = strongly disagree, 2 = disagree somewhat, 3 = neutral, 4 = agree somewhat, 5 = strongly agree)

Teacher Control

At this school, how much control do you feel teachers have in their classrooms over each of the following areas of planning and teaching: (a) Selecting textbooks and other instructional materials; (b) Selecting teaching techniques; (c) Evaluating and grading students; (d) Disciplining students; (e) Determining the amount of homework to be assigned; (f) Student promotion and retention. (1 = no control, 2 = little control, 3 = some control, 4 = considerable control, 5 = strong control)

Teacher Influence At this school, how much influence do you think teachers have over school policy in each of the following areas: (a) Evaluating teachers; (b) Setting discipline policy; (c) Hiring new full-time teachers; (d) Deciding how the school budget will be spent; (e) Determining the content of in-service programs. (1 = no influence, 2 = little influence, 3 = some influence, 4 = considerable influence, 5 = strong influence).