Achievement Results from the B.C. Mathematics Assessment*

David F. Robitaille and James M. Sherrill
university of british columbia

Les résultats d'une évaluation des connaissances en mathématiques faite à l'échelle provinciale démontrent que les aptitudes de base en arithmétique chez les étudiants inscrits en 4e, 8e et 12e années sont généalement satisfaisantes et qu'ils sont suffisamment familiers avec la notation et la terminologie mathématique. Les résultats des problèmes résolus par des élèves de 4e et de 8e années furent relativement bons, mais pas en 12e année. Des résultats moins que satisfaisants furent obtenus pour les questions se rapportant aux concepts d'ordre logique et pour celles se rapportant à la géométrie et aux mesures. Les données semblent aussi démontrer l'existence d'une relation entre le succès des élèves et des facteurs tels que l'âge, le sexe, le degré d'instruction des parents et l'origine ethnique.

In March 1977 over 100,000 students enrolled in Grades 4, 8, and 12 in the public schools in British Columbia took part in an assessment of mathematics learning conducted by the Learning Assessment Branch of the Ministry of Education. The authors, assisted by two classroom teachers, conducted the study for the ministry.

As with other projects of its kind in North America, the British Columbia Learning Assessment Program is designed to meet several objectives. In the first place, the program is intended to inform the members of the profession and the general public of some of the strengths and weaknesses of the public school system in the province. Secondly, the information produced should be helpful in the processes of curriculum development and revision. Thirdly, the results could be used to influence the direction of change in teacher-education programs and in the allocation of resources at both the provincial and local school district levels. Furthermore, the data should be of considerable interest to educational researchers wishing to pursue follow-up studies.

THE ITEM SPECIFICATION MODEL

The topics to be assessed were restricted to those which most informed observers would agree were among the essential concepts and skills of mathematics at the three levels selected for testing. The Grade 4 test was designed as an end-of-primary-education test in mathematics, so its

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content was restricted to the curriculum of Grades 1 to 3. Similarly, the Grade 8 test was designed as an end-of-elementary-education test, and its content was limited to the curriculum of Grades 4 to 7. The Grade 12 test, which was administered to all students at that level, regardless of their mathematics backgrounds, was designed to measure the extent to which students completing secondary school had mastered certain essential concepts and skills of mathematics.

The Item Specification Model shown in Figure 1 was developed to aid in the process of constructing test items. It portrays graphically the three major dimensions of the Mathematics Assessment: three grades (4, 8, and 12), four content areas (Number and Operation, Measurement, Geometry, and Algebraic Concepts), and three levels of cognitive behavior (Computation and Knowledge, Comprehension, and Applications).

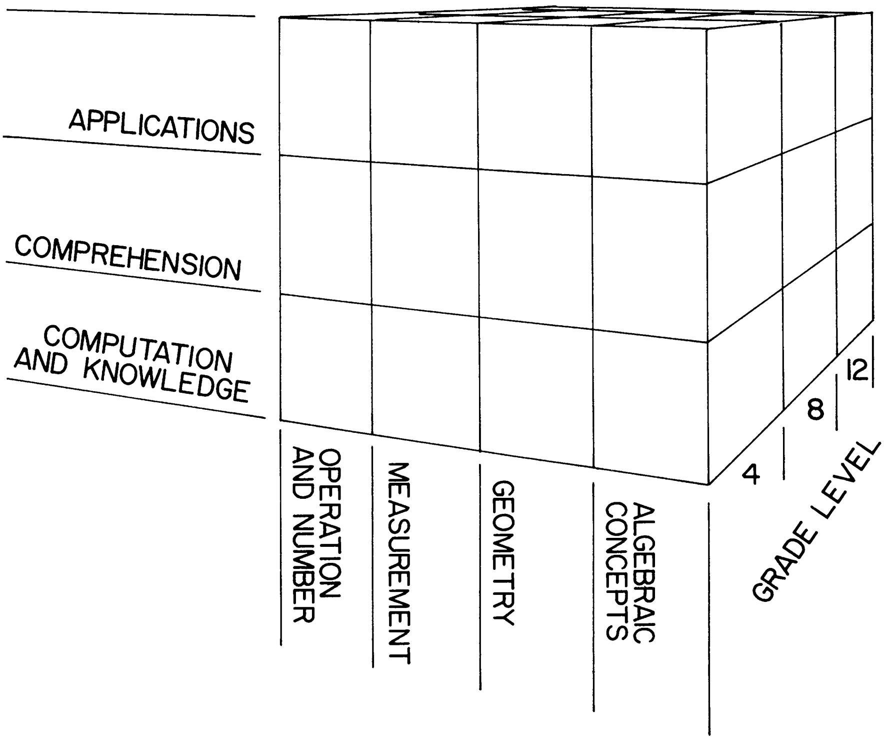


FIGURE 1

Item Specification Model

The four content areas which were tested include many of the major topics in contemporary mathematics curricula. Items classified in the Number and Operation category dealt with the nature and properties of whole numbers, integers, rational numbers, and real numbers as well as with arithmetic operations and their properties. The Measurement category included topics such as selecting the most appropriate unit of measurement, familiarity with SI metric units, area, perimeter, volume, angular measure, and scale drawing. Topics in the Geometry category dealt with the identification of geometric figures, classification of angles and triangles, naming the parts of a circle, and applying the Theorem of Pythagoras. Items from the Algebraic Concepts category were concerned with interpreting graphs, writing algebraic expressions, simplifying and evaluating polynomials, solving linear and quadratic equations, finding slopes of lines, and solving simultaneous equations.

The third dimension of the Model consisted of three levels or Domains of cognitive behavior. The Computation and Knowledge Domain encompassed such areas as knowledge of facts, knowledge of terminology, and ability to use standard algorithms. The Comprehension Domain included understanding of concepts, understanding of principles, rules, and generalizations, ability to transform problem elements from one mode to another, and ability to read and interpret problems. Ability to solve routine problems, ability to analyze data, and ability to recognize patterns were included in the Applications Domain.

The Domain dimension of the Model is hierarchical. Solution of items in the Applications Domain requires cognitive behaviors which are more sophisticated than those required to solve items in the Comprehension Domain. Similarly, solution of items in the Comprehension Domain requires cognitive behaviors which are more sophisticated than those used in solving items in the Computation and Knowledge Domain.

TEST ADMINISTRATION

The Grade 8 and Grade 12 tests consisted exclusively of multiple-choice items, with each item having five choices: four possible answers to the question and "I don't know." The Grade 8 test contained 60 items and the Grade 12 test contained 72 items. Forty-one items were common to both tests.

The Grade 4 test consisted of three parts. The first was a test of students' mastery of number facts for addition, subtraction, multiplication, and division, and students were given four minutes to complete these 24 open-ended items. Part II of the test contained 15 open-ended computation items, and Part III contained 30 multiple-choice items.

The tests were mailed to all of the public schools in the province and administered by the classroom teachers. One and one-half hours were allotted for students to take the test, this having been shown to be sufficient time for virtually all students to complete the test. The test booklets and completed OMR cards were returned by mail for processing.

As is shown in Table 1, the completion rate for the tests varied from a low of 71.1% at the Grade 12 level to 90.1% at Grade 8, and 96.5% at Grade 4.

TABLE 1

Rate of Completion of Mathematics Assessment Tests

Grade/Year	*Enrollment as of February 1977**	*Number of Completed Tests*	*Percent Completed*
4	36,540	35,277	96.5
8	46,808	42,250	90.1
12	32,532	23,136	71.1
Overall:	115,880	100,663	86.9

*Data supplied by the B.C. Ministry of Education.

Of the three grade level groups, Grade 12 was the only one which had a lower rate of return than might have been attributable to normal absenteeism. In addition, concern was expressed by some educators as well as in the press that senior secondary students might not have taken the test seriously.

The National Assessment of Educational Progress in the U.S.A. encountered a similar problem of non-participation by senior secondary students. The results of a follow-up study (Rogers, et al., 1977) indicate that, although the non-participating group differed in composition from the participating group, the failure of such students to take the test did not greatly inflate the results.

As regards the extent of frivolous response, examination of the B.C. data showed that less than 0.1% of Grade 12 students had responded by systematically choosing the same response choice to each item or by selecting an obvious repetitive pattern of responses: e.g. A,B,C,A,B,C. . . . Moreover, less than 1% of the respondents obtained a total test score of less than 20%, the rate of correct response which could have been expected from a random selection of the alternatives.

INTERPRETATION OF RESULTS

The student tests at all three levels were machine-scored, and the percentage of students who obtained the correct answer was computed for each item. For each multiple-choice item, the percentage selecting each incorrect choice was also calculated.

Three 15-member Interpretation Panels, one for each grade level tested, were appointed by the Ministry of Education. Each Panel included seven teachers of mathematics at the grade level in question, two supervisors of instruction, two teacher educators, two school trustees, and two members of the public at large. Following a procedure designed to assist them in arriving at a consensus, the panelists examined the results for each item and assigned to each result one of the five ratings which are listed below along with their abbreviated designations:

Strength	ST
Very Satisfactory	VS
Satisfactory	S
Marginally Satisfactory	MS
Weakness	W

A summary of the ratings given by the Panels is shown in Table 2.

TABLE 2

Summary of Ratings Given by Interpretation Panel

	Number Of Items	*W*	*Number of Items by Category* *MS*	*S*	*VS*	*ST*
Grade 4	69	4	12	17	25	11
Grade 8	60	6	15	25	11	3
Grade 12	72*	7	15	25	21	3

*The Grade 12 Interpretation Panel declined to rate performance on one item.

The general picture which emerges from the Panel ratings is satisfactory. While it is true, on the one hand, that a number of weaknesses were noted and that attention should be paid to these areas, it also seems clear that students are learning many of the basic concepts and skills of mathematics which are expected of them. At the Grade 4 level, 77% of the item results were judged to be satisfactory or better. At the Grade 8 level, 65% were so judged; and, at the Grade 12 level, 69%.

TEST RESULTS - GRADE 4

The three Domains on the Grade 4 test were subdivided into 10 objectives, and the Interpretation Panel was pleased with student performance on six of these. The six were Comprehension of Number Properties (90.0%), Mastery of Number Facts (88.3%), Addition of Whole Numbers (87.0%), Solution of Practical Problems (78.0%), Knowledge of Notation and Terminology (74.8%), and Solution of Computational Problems (64.7%). On the other hand, the Panel was less satisfied with student performance on Subtraction of Whole Numbers (74.8%), Comprehension of Fraction Concepts (57.0%), and Comprehension of Measurement Concepts (54.0%).

In the Computation and Knowledge Domain, the Interpretation Panel judged students' performance on the 12 number fact items for addition and subtraction to be a Strength. They rated as Very Satisfactory the achievement on multiplication facts, and as Satisfactory the performance on division facts. Since the number fact section of the test was timed, and since the six division fact items were the last items in the number fact section of the test, it was not possible to tell whether the relatively high rates of missing responses on these six items (between 13% and 21%) were due to the difficulty of these facts or to a lack of time to complete them.

Elsewhere in the Computation and Knowledge Domain, a Weakness was noted on an item dealing with subtraction of whole numbers. Only 56% of the students obtained the correct answer to the item,

$$\begin{array}{r} 1054 \\ -\ 865 \\ \hline \end{array}$$

which required them to perform a subtraction exercise involving multiple regroupings (i.e. "borrowing"), and including a zero in the minuend. None of the subtraction with regrouping results was rated any higher than Marginally Satisfactory, and this suggests that more attention should be paid to developing students' understanding of place value concepts, of techniques for regrouping, and of applying these concepts to the operation of subtraction.

Grade 4 results in the Comprehension Domain were less satisfactory than in either of the other two Domains. Whereas performance of Understanding of Number Properties was very good, performance on the other three objectives in this Domain was not. Two item results from the Comprehension of Measurement Concepts Objective were rated as Weaknesses. The items, which are shown in Figure 2, assessed students' familiarity with temperature measured in degrees Celsius and weight (mass)[1] in kilograms.

According to the B.C. Curriculum Guide for Mathematics, these students should have been taught the metric system of measurement and only the metric system since they entered school. In spite of this, only 32% chose the appropriate temperature for a sunny summer day, and only 25% were able to select the most likely mass for a 10-year-old boy. In each case only one answer choice was even a reasonable candidate for selection. The other three were clearly incorrect to anyone familiar with the units involved.

Performance on Comprehension of Fraction Concepts was also rather weak. On Item 58 (See Figure 3), where 60% of the students responded correctly, performance was Marginally Satisfactory, and on Item 60, where 54% answered correctly, it was Weak.

(64) A ten-year-old boy is likely to weigh:

	Percent
35 grams	9
75 grams	22
35 kilograms	25
75 kilograms	33
I don't know	8
No response	2

(65) The temperature on a sunny summer day would most likely be:

	Percent
5° Celsius	5
25° Celsius	32
55° Celsius	27
85° Celsius	29
I don't know	6
No response	1

FIGURE 2

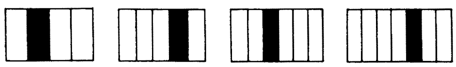
Grade 4 — Items 64 and 65

(58) Which group of dots is one-half (½) shaded?

 I don't know

21 *60* 8 2 7

(60) Which box is one-fifth (1/5) shaded?

 I don't know

7 *54* 15 17 5

FIGURE 3

Grade 4 — Items 58 and 60

Introductory fraction concepts are part of the standard mathematics curriculum at the primary-grade level, and students are expected to learn about fractions both as parts of wholes and as parts of sets. On the other hand, these results may mean that such concepts are too advanced for many children of this age. That is, because of their level of intellectual development, many of these children may not be capable of grasping fraction concepts. Curriculum developers and educational researchers should give the question of when to introduce fraction concepts to children their immediate attention.

The results from the items in the Applications Domain were commendable. Problem-solving is the most difficult topic in the mathematics curriculum to teach, and it is also a difficult topic for students to learn. In spite of this, no item results were rated as Weaknesses, and one was rated a Strength. This latter item dealt with students' ability to read information from a graph.

TEST RESULTS - GRADE 8

The test administered to Grade 8 students contained 60 items which were grouped under 12 objectives and three Domains. The proportion of Grade 8 item results which were rated by the Grade 8 Interpretation Panel as being Very Satisfactory or a Strength was lower than the corresponding proportions of ratings given by the other Panels at their grade levels. Correspondingly, a higher proportion of Grade 8 item results were rated either Weak or Marginally Satisfactory, although the difference between the Grade 8 and Grade 12 results in this regard was slight. The overall picture, while not being one of weakness, seems to show performance at this level to be the least satisfactory of the three levels tested.

In the Computation and Knowledge Domain, the strongest performances were recorded on items dealing with Computation with Whole Numbers (83.8%) and Knowledge of Notation and Terminology (70.8%). The areas of Computation with Fractions (68.2%), Computation with Decimals (67.6%), Knowledge of Geometric Facts (63.2%), and Equivalent Forms of Rational Numbers (59.0%) were weaker in varying degree, with the last-named being weakest of all.

Grade 8 students' ability to write a number expressed as a fraction, decimal, or percent in either of the other forms was less than satisfactory. For example, only 38% of students answered correctly Item 31 (See Figure 4), and this performance was rated a Weakness.

In the Comprehension Domain, Strengths were outnumbered by Weaknesses 4 to 2. Performance was relatively good on items dealing with Comprehension of Measurement Concepts (69.4%), but somewhat weaker on Comprehension of Geometric Concepts (53.8%) and on Comprehension of Number Concepts (53.5%).

Two of the results rated as Weaknesses in this Domain dealt with fraction concepts. One of the items (See Figure 5) required students to use

31. Written as a decimal, $\frac{1}{8}$ =

		Percent
A)	0.12	6
B)	0.8	41
C)	0.125	38
D)	0.18	8
E)	I don't know	5
	No response	1

FIGURE 4

Grade 8 — Item 31

the concept of the meaning of a fraction while the other required them to order fractions. These same items appeared on the Grade 12 test, and the performance of students at that level was also considered to be weak on both of these items. In Figure 5, the results obtained at both levels are displayed.

18. There are 13 boys and 15 girls in a group. What fraction of the group is boys?

		Percent Grade 8	Grade 12
A)	$\frac{15}{28}$	5	5
B)	$\frac{13}{15}$	53	38
C)	$\frac{15}{13}$	6	4
D)	$\frac{13}{28}$	32	51
E)	I don't know	2	1
	No response	1	1

47. Which number is largest?

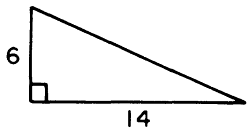
		Percent Grade 8	Grade 12
A)	$\frac{2}{3}$	38	21
B)	$\frac{4}{5}$	29	59
C)	$\frac{3}{4}$	16	13
D)	$\frac{5}{8}$	12	5
E)	I don't know	2	1
	No response	1	1

FIGURE 5

Grade 8 — Items 18 and 47

Performance on items dealing with Comprehension of Geometric Concepts was disappointing, with none of the four item results being rated higher than Satisfactory, and two of them being rated below that level. The weakest performance was registered on an item dealing with the area of a right triangle (see Figure 6). Almost twice as many students chose 84, the area of the corresponding rectangle, as chose the correct response.

52. Find the area of this right triangle.



		Percent
A)	42	24
B)	20	18
C)	84	42
D)	21	4
E)	I don't know	11
	No response	1

FIGURE 6

Grade 8 — Item 52

TEST RESULTS - GRADE 12

The Grade 12 test contained 72 items grouped under 11 Objectives and three Domains. The test was administered to all students enrolled in Grade 12 whether or not they were taking a mathematics course at the time. It was not intended as a measure of students' mastery of the content of any particular course, such as Math 10, Math 11, or Math 12; but rather as a test of students' mastery of a number of essential mathematical concepts and skills which, for the most part, all students could be expected to have mastered upon completion of secondary school. The mathematics content of each item was selected from the Grade 10 level or below. For these reasons, the results summarized here should not be used to characterize the success of any particular course, nor should they be interpreted as indicators of students' preparedness for post-secondary courses in mathematics.

Three Grade 12 item results from the Computation and Knowledge Domain were rated as Strengths and none as Weaknesses. Performances on the four objectives in this Domain were as follows: Computation with Fractions (83.3%), Computation with Decimals (80.2%), Knowledge of Notation and Terminology (75.7%), and Knowledge of Other Algorithms (72.7%).

Two of the Grade 12 results from the Comprehension Domain were rated as Weaknesses. One, which was discussed earlier and which is shown in Figure 5, dealt with fraction concepts while the other was concerned with the metric unit of mass.

Results on five of the 18 problem-solving items from the Applications Domain were rated as Weaknesses, and four others as Marginally Satisfactory. None were rated as Strengths. Four of the weak results occurred on problems dealing with geometry and measurement, one of these being Item 47, which is shown in Figure 5. The fifth weak result was noted on an item dealing with the use of the simple interest formula

$$i = Prt$$

which was given along with the item shown in Figure 7.

69. Find the *principal*, if the interest received after two years at an annual rate of 6% is $60.

		Percent
A)	$2000	9
B)	$5000	7
C)	$ 500	48
D)	$ 720	17
E)	I don't know	16
	No response	2

FIGURE 7
Grade 12 — Item 69

The Grade 12 results in the Applications Domain were disappointing. They indicate that many students are unable to apply the computational skills which they have learned to certain types of problems. This appears to be especially true for problems involving applications of geometry and measurement ideas.

One of the major factors to be borne in mind in considering the Grade 12 results is that these students vary considerably in the nature and extent of their backgrounds in mathematics. About 15% of these students, the Math 10 group, had taken no courses in mathematics beyond Mathematics 10, the last course required of all students in B.C. About 35%, the Math 12 group, had taken or were taking Mathematics 12, while the remainder, the Math 11 group, had taken no courses beyond Mathematics 11.

On the average, the Math 12 group's results were approximately 20% higher than those of the Math 11 group which were, in turn, about 15% higher than those of the Math 10 group. As a matter of fact, the Math 10 group achieved at about the same level as Grade 8 students on those items which were common to both tests. The relative performances of the Math 10, Math 11, and Math 12 groups are shown graphically in Figure 8 below.

REPORTING CATEGORIES

Each test booklet included a number of questions which sought information about various aspects of the students' backgrounds which were considered to be related to achievement in mathematics. Although it is not possible to impute cause-and-effect relationships on the basis of such data, the results do reveal several trends which could form the basis for future investigations. A few of the more interesting of such reporting categories are discussed below.

Age: Among Grade 4 students who were 9 years old when they took the test, those who were born in the first quarter of the year (1967) did better on all of the objectives tested than those born between April and December of the same year. That is, among the 9-year-olds in Grade 4, the older students outperformed the younger ones.

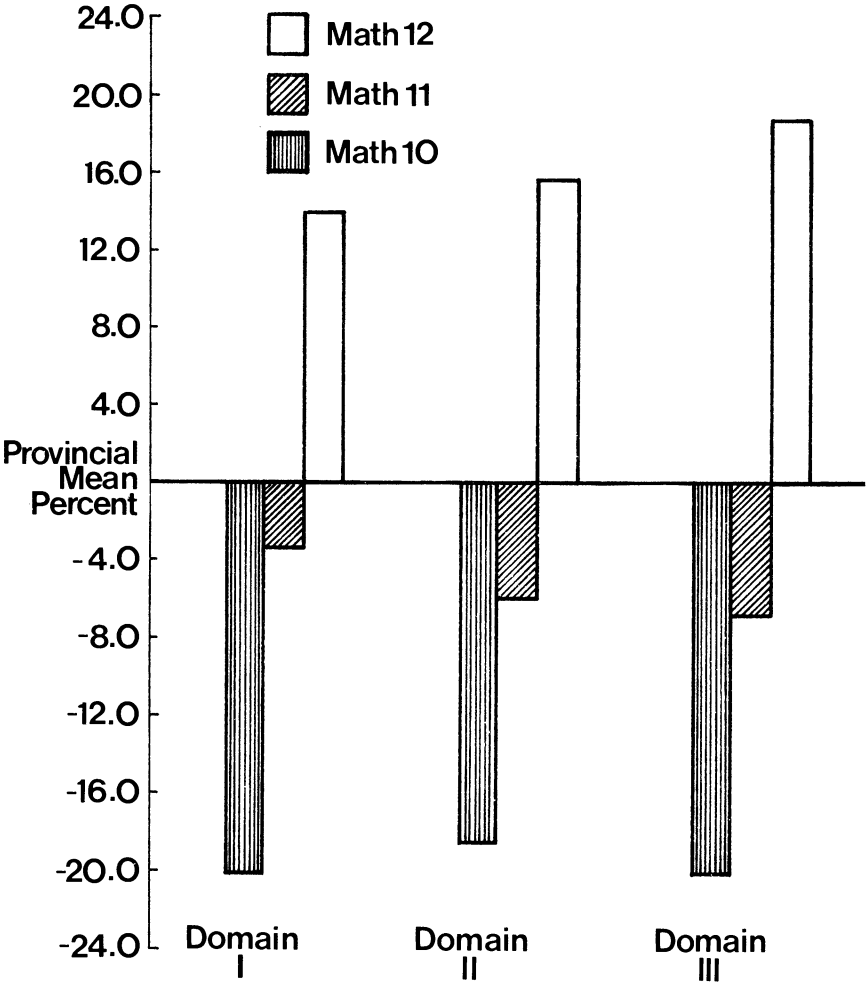


FIGURE 8

Grade 12 Results by Students' Background in Mathematics

Similar results have been reported elsewhere (Callahan and Glennon, 1975) among younger children. What is most interesting in the present case is that the differences appear to persist through the end of Grade 4. No comparable differences were found at the Grade 8 level.

Sex: A comparison of the results obtained on each Domain by males and by females is presented in Figure 9. At all three levels, boys performed somewhat better than girls in Domains 2 and 3. Girls' performance was superior to that of boys only in the Computation and Knowledge Domain in Grades 8 and 12. Similar differences were noted when comparisons at the Grade 12 level were made with the mathematics background variable controlled.

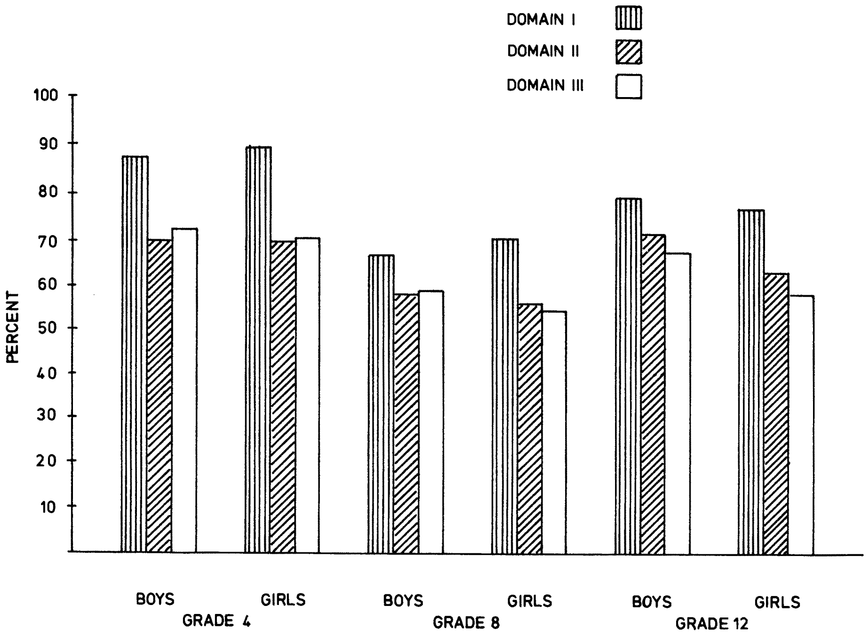


FIGURE 9

Domain Results by Sex

These results coincide with those reported in other studies such as the National Assessment of Educational Progress (NAEP, 1975) and the National Longitudinal Study of Mathematics Ability (School Mathematics Study Group). Although the matter is currently the subject of some debate (Fennema, 1977), the results generally indicate that females outperform males on mathematics objectives which require cognitive behaviors from the lower end of the scale, whereas males outperform females on objectives which require higher order cognitive behaviors.

Mother Tongue and Country of Origin: Among Grade 8 and Grade 12 students, the group consisting of those who had not been born in Canada and for whom English was not the first language outperformed the Canadian-born, English-as-a-first-language group on all three Domains. This situation was reversed at the Grade 4 level.

Parental Level of Schooling: Test results for Grade 12 students showed a general pattern of increase in student achievement associated with an increase in the highest educational level attained by parents. Separate analyses were made using father's and mother's educational levels, with virtually identical results. For that reason, only the relationship between father's educational level and student achievement is shown in Figure 10.

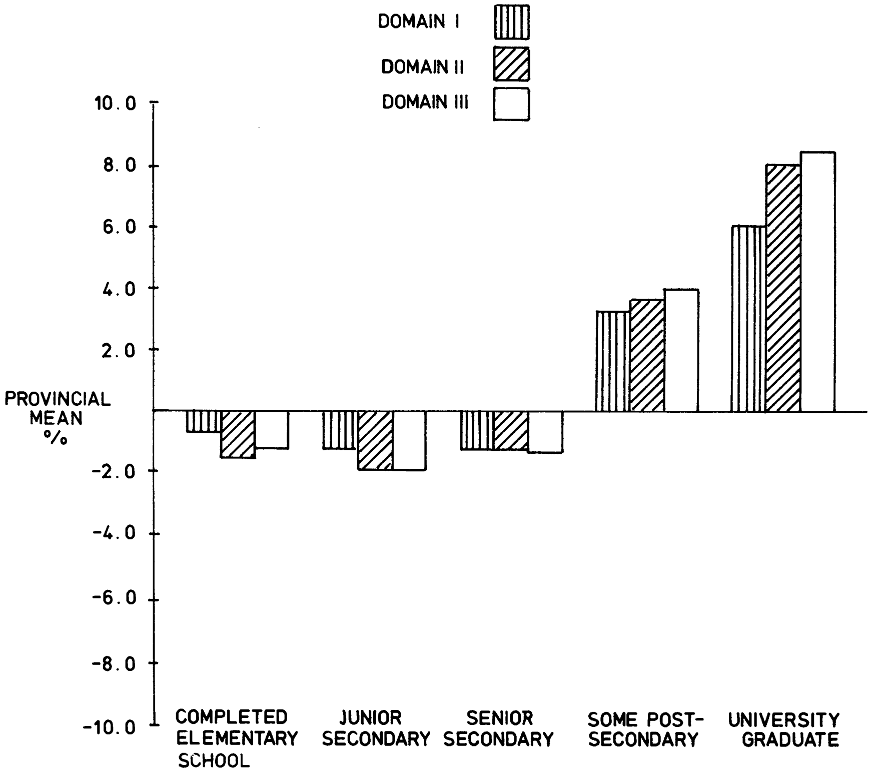


FIGURE 10

Grade 12 Results by Father's Educational Level

CONCLUSION

The results of the B.C. Mathematics Assessment reveal a number of areas where, in the opinion of the Interpretation Panels, student achievement is relatively good. In addition, the Panels noted some areas where improvement is needed. Among the former are computational skills, knowledge of terminology, and, at the Grade 4 and Grade 8 levels, problem-solving. Areas requiring improvement include comprehension of fraction concepts, comprehension of geometry and measurement concepts, and, particularly at the senior secondary level, problem-solving.

NOTE

1 The word "mass" was not used on the tests although it is the correct term. It was deemed better to use the more familiar term "weight."

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David F. Robitaille and James M. Sherrill are Associate Professors of Mathematics Education in the Faculty of Education, University of British Columbia, Vancouver, British Columbia v6T 1W5.