

# **A Formula for Success?**

## **An Examination of Factors Contributing to Quebec Students' Strong Achievement in Mathematics**

Ilona I. Vashchyshyn  
*University of Saskatchewan*

Egan J. Chernoff  
*University of Saskatchewan*

### **Abstract**

As the only province having achieved significantly above the Canadian average in the latest PISA assessment, and with an average score that was surpassed by only five other participating countries, Quebec has recently taken centre stage as Canada's superstar in the teaching and learning of mathematics. However, there has been relatively little research surrounding why Quebec students have been consistently successful in their mathematical endeavours. In this essay, the authors examine several possible influences, including ample opportunities for students to participate in recreational mathematics activities, an emphasis on problem solving, intensive teacher education programs, and active mathematics teacher associations. Our aim is to begin a conversation surrounding the following question: What can we, as mathematics teachers, learn from our neighbours in *la belle province*?

*Keywords:* mathematics education, teacher education, teacher organizations, problem solving, recreational mathematics

## **Résumé**

Seule province à avoir surpassé la moyenne canadienne à l'examen PISA, et ce, avec un résultat dépassé par seulement cinq autres pays, le Québec occupe depuis quelques années le devant de la scène canadienne en tant que vedette de l'enseignement et de l'apprentissage des mathématiques. Or, il semble que jusqu'ici, les chercheurs se soient peu demandé pourquoi les étudiants québécois réussissent à soutenir un aussi haut niveau de performance dans ce domaine. Dans cet article, les auteurs examinent plusieurs influences possibles, y compris maintes occasions pour les élèves de participer aux activités mathématiques récréatives; l'accent fort que les enseignants québécois mettent sur la résolution de problèmes; des programmes de formation intensifs; et des associations actives de professeurs de mathématiques. Notre but est d'engager une conversation sur la question suivante: qu'est-ce que nous, en tant que professeurs de mathématiques, pouvons apprendre de nos voisins de la belle province?

*Mots-clés :* enseignement des mathématiques, formation des enseignants, associations des enseignants, résolution de problèmes, mathématiques récréatives

## Introduction

In 2012, 65 countries participated in the Programme for International Student Assessment (PISA), including approximately 21,000 Canadian students from about 900 schools across the 10 provinces (Brochu, Deussing, Houme, & Chuy, 2012). Since its inauguration in 2000, PISA reports on the mathematical, reading, and scientific literacy of a sample of 15-year-old students in each participating country every three years, with one of these domains selected for more detailed study at each cycle. Mathematical literacy (as defined by PISA) was the focus of the 2012 assessment, and that year, PISA assessed three processes related to the mathematical domain: formulating situations mathematically; employing mathematical concepts, facts, procedures, and reasoning; and interpreting, applying, and evaluating mathematical outcomes. Overall, Canadian students achieved above the Organisation for Economic Co-operation and Development (OECD) average, surpassed by only nine other participating countries (Brochu et al., 2012). According to PISA, “Canadian students achieved strong results in each of the three [mathematical] processes assessed” (Brochu et al., 2012, p. 22). So why did John Manley, the CEO and president of the Canadian Council of Chief Executives (as cited in Editorial, 2013), call the results “a national emergency?”

While it is true that, according to PISA results and other comparable mathematics assessments conducted over a nine-year span, the performance of Canadian 15-year-olds declined by a small, but statistically significant amount, Canada was not the only country that experienced a decline in scores: A decrease in average achievement was also observed in the Netherlands, Finland, and Belgium (Brochu et al., 2012). Among “high-performing” countries, only Macau-China, Poland, and Germany improved in mathematics over the past four PISA cycles. And yet, it appears that the Canadian population took the results to heart, with notable newspapers such as *The Globe and Mail* suggesting that Canada is doing no less than “failing to effectively teach our students math” (Editorial, 2013).

One province, however, stood out from the rest. Only students in Quebec achieved significantly above the Canadian average in each of the three mathematical processes assessed by PISA, and impressively so: their average score was surpassed by only five other participating countries (Brochu et al., 2012). And so, almost overnight, Quebec became Canada’s superstar in the teaching and learning of mathematics, with all eyes turning to *la*

*belle province* in the hopes of discovering the “formula to better nationwide math scores” (Peritz, 2013). Many explanations have been offered for the discrepancy in scores, with the media (see, for example, Peritz, 2013; Editorial, 2013; Editorial, 2014) often framing the issue as a divide between “traditional” and “reform” (or “discovery”) mathematics (see, for example, Schoenfeld, 2004 for a history of the debate). For instance, on December 3, 2013, an article in *The Globe and Mail* declared that “Quebec schools, when compared with those in the rest of Canada, use more memorization and rote learning” (methods associated with the “traditional” approach of teaching mathematics) and have largely “ignored the fad” of discovery mathematics (Editorial, 2013). Strangely, only three days later, an article in the same newspaper claimed that Quebec *favours* “discovery learning,” which is “meant to encourage kids to learn concepts by solving problems rather than memorizing rules and equations” (Peritz, 2013). Such examples suggest that in the public sphere, the success of Quebec students has been largely misunderstood and that the issue has only been further obscured by popular media outlets.

So what *is* Quebec doing right? First of all, it should be acknowledged that Quebec is not necessarily a “paradise” when it comes to the teaching of mathematics, as Laurent Theis, a mathematics education researcher at the University of Sherbrook explains (personal communication, January 19, 2015); moreover, as in all other provinces, the mathematics education program is not universally accepted (Dionne, 2007). However, Theis and other researchers in the domain (e.g., Dionne, 2007; Peritz, 2013) also confirm that the province *does* have reason to be proud of its students, who have consistently performed strongly in international assessments such as PISA and the Trends in International Mathematics and Science Study (TIMSS). Although many myths and controversies surround the issue, it is possible to identify some likely contributions to the province’s success from the available research. Several influences that have likely had a positive effect on the teaching and learning of mathematics in Quebec will be examined in this article, including ample opportunities for students to participate in recreational mathematics, an emphasis on problem solving in mathematics classrooms, intensive and comprehensive teacher education programs, and active mathematics teacher associations that support primary and secondary school educators. (A discussion of social and structural influences, such as the possible positive effect of competition between public and private schools, is beyond the scope of this article; such factors have been examined in, e.g., Richards [2014a, 2014b].) Rather, the present discussion focuses on the effective aspects

of mathematics education in Quebec that may be more readily adapted to the other Canadian provinces. In other words: What can teachers of mathematics across Canada learn from their neighbours in *la belle province*?

### **Recreational Mathematics in Quebec**

One characteristic feature of the “life” of mathematics in Quebec is that students at all grade levels have many opportunities to practise their developing skills in mathematics, and particularly in problem solving, *outside* of traditional classroom tasks through “recreational mathematics” activities organized by a multitude of provincial mathematics associations. Since the 1980s, associations of mathematics teachers in Quebec have established a considerably wide array of mathematical contests with the intention of creating opportunities for students to exercise their imagination and creativity while developing their problem-solving skills (Dionne, 2007). Among the more popular of these is Opti-Math, an annual contest for all secondary-level students organized by the Groupe des responsables de la mathématique au secondaire (GRMS) since 1988. The Mathematical Association of Quebec (Association Mathématique du Québec: AMQ) also organizes an annual contest for secondary and collegiate level students in Quebec, which is aimed more at “elite” mathematics students.

International contests are also common in the province. For instance, the Association Québécoise des Jeux Mathématiques (AQJM), whose objective is described on its website as to promote mathematics to *all*—to primary, secondary, and university students, as well as to the general public—annually oversees and promotes the International Competition for Mathematical and Logical Games (Le Championnat international des jeux mathématiques) in Quebec, with great success. The AQJM also organizes other recreational mathematics events throughout the year, including a series of mathematical games and activities at the Grande Bibliothèque de Montréal. According to the AQJM website, activities include mathematical “magic” tricks, puzzles, games, and riddles, which align with their goal of revealing the fun, attractive side of mathematics. Although the correlation between achievement in mathematics and participation in mathematical games, puzzles, and contests has not been firmly established, de Guzman (1990) does suggest that “good games and puzzles can avoid the effect of the psychological blocks that straight

mathematical presentations tend to cause...very often because of previous unpleasant mathematical experiences” (p. 365). Indeed, he contends that “many profound ideas of the greatest mathematicians could be traced down to their involvement in this kind of ludic thinking [i.e., in mathematical puzzles and games],” citing, among other examples, Fermat’s deep and extensive investigations on magic squares (de Guzman, 1990, p. 366).

The rate of participation in the aforementioned contests is impressive: For instance, the Opti-Math website boasts that a total of 265 schools registered to participate in the 2015 Opti-Math contest (most from Quebec, but some from New Brunswick and Alberta). In 2015, 1,364 of the top solutions to the given problems were sent in for correction; considering that this represents only a small percentage of all students who attempted the problems, the GRMS suggests that, in fact, several thousand students participate in the contest annually. The rate of participation in the International Competition for Mathematical and Logical Games is even more impressive: According to a representative of the AQJM, nearly 18,000 students in Quebec participated in the competition in the last two years. By comparison, according to estimates provided to the authors by contest representatives, the more popular mathematical competitions in other provinces (e.g., the New Brunswick Math Competition, the Ontario-based Math@Mac Online Competition, and the Calgary Elementary School Math Contest) draw in a maximum of approximately 2,000 students per year (although it is encouraging to note that some of these representatives reported increasing participation rates over the past few years).

What seems to distinguish the culture of mathematical contests in Quebec is that, in general, they seek to attract the “average” mathematics student, rather than only the elite. Massive participation is indeed one of the goals of the Opti-Math initiative, which emphasizes participation rather than performance. According to the official contest guide, the slogan is not “Que le meilleur gagne” (May the best win) but rather “Que le plus grand nombre participe et s’améliore en résolution de problèmes” (May the greatest number of students possible participate and improve their problem-solving skills)—which, admittedly, is not as memorable, but it certainly helps to diffuse the anxiety that may be associated with some mathematical competitions and pinpoints the organizers’ priorities (Le Comité central des Concours Opti-Math du GRMS, 2015). As for the impact on educators, Dionne (2007) suggests that contests such as these have led to the spread of ideas like the “open problem” and to problem solving becoming established in the culture of primary schools in Quebec. As previously mentioned, the impact of recreational

mathematics activities in terms of educational achievement on *students* has so far been difficult to establish. According to Hogle (1996), the most difficult issue in the assessment of games as cognitive tools is that they tend to foster the learning of implicit, rather than explicit knowledge. Implicit knowledge, suggests Hogle, is not necessarily reflected in students' ability to answer written questions; however, this does not mean that real benefits have not been achieved.

## **A Focus on Problem Solving**

In addition to ample opportunities for students to participate in recreational mathematics activities, an emphasis on problem solving is likely another positive influence on Quebec students' performance on international mathematics assessments such as PISA. According to the latest provincial mathematics curriculum, students in Quebec are to develop three particular competencies over the course of their schooling: (1) solving situational problems, (2) using mathematical reasoning, and (3) communicating via "mathematical language" (Ministère de l'Éducation du Québec [MEQ], 2004). This suggests, as Lajoie and Bednarz (2012) affirm, that problem solving is at the heart of mathematics education in Quebec.

While a rigorous definition of "problem solving" is beyond the scope of this article, a brief summary of common conceptions may be helpful. George Polya (whose 1945 book *How to Solve It* is arguably the Bible of mathematical problem solving) incorporated the concept of novelty in his definition, describing mathematical problem solving as finding a way around an unfamiliar difficulty or obstacle (1949/1980). Others (e.g., Lester & Kehle, 2003) add that reasoning is a critical component of mathematical problem solving. Likewise, the OECD, creator of the PISA assessment, considers problem-solving skills to be "an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious" (OECD, 2013, p. 122). Problem-solving competency, according to the OECD, involves "far more than the basic reproduction of accumulated knowledge" (p. 122). Indeed, the MEQ describes mathematical problem-solving competency as the capacity to find a logical solution to a problem that corresponds to the following conditions: the situation has not been encountered during previous lessons; the application of a combination of

rules and principles that may *or may not* have been previously acquired by the student is required in order to solve the problem; and the product, or its expected form, has not been previously presented (MEQ, 2004). In other words, the MEQ encourages teachers to build students' capacity to deal with novelty and uncertainty in the mathematical domain, which, considering the definitions provided by the OECD above, has likely contributed to their success on past PISA assessments.

However, according to Jonnaert (as cited in Fagnant & Vassis, 2010), problem solving is viewed by educators in Quebec not only as a way to apply mathematical notions or as a subject for study in and of itself but also as a pedagogical tool. In the latter case, rather than being studied at the *end* of a unit, problem solving is meant to be used during *all* stages of learning, including as a means of introducing and exploring new mathematical concepts, properties, algorithms, and so on (Bednarz, 2002; Lajoie & Bednarz, 2014, 2012). This approach is not new in the province. As Lajoie and Bednarz (2014, 2012) explain in their account of the evolution of problem solving in mathematics education in Quebec, problem solving has served these three roles in mathematics classrooms in the province since the 1970s. They add that the role of the teacher during the problem-solving process is also considered to be critical. Rather early on during this evolution, it was established that the mathematics teacher should aim to *facilitate* the process, rather than to be a “demonstrator of solutions.” For instance, according to the *Fascicule K du Guide pédagogique*, a problem-solving guide for primary school teachers published in 1988 by the Quebec Ministry of Education, the teacher should aim for a maximum degree of contribution on the part of the student (Lajoie & Bednarz, 2012). As Lajoie and Bednarz (2012) explain, the principle extends not only to finding the solution but also to creating the problems, finding the data, and/or choosing the problems to be solved.

It is interesting to note that this situation parallels that of Finland, which has consistently been a top performer in PISA and other international assessments and has therefore garnered considerable press attention in Canada and around the world. According to Dossey and Wu (2013), since 1985, the focus in school mathematics in Finland shifted from an emphasis on basic concepts and structure to one emphasizing problem solving, application, and everyday uses of mathematics. This shift was accompanied by professional development for teachers on teaching through problem solving and the use of projects to involve students in using mathematics to solve contextualized problems from everyday settings (Dossey & Wu, 2013). In connection to the next section, it should also

be noted that teachers in Finland generally have a more advanced education than their peers in most countries, and that teacher preparation programs strive to strike a balance between content knowledge and didactics, or pedagogical content knowledge (Dossey & Wu, 2013; Tirri, 2014). (We will see shortly that this parallels the situation in Quebec.) Certainly, a multitude of factors other than teacher education, including sociological and historical factors, have contributed to the academic success of Finnish students (Andrews, Ryve, Hemmi, & Sayers, 2014; Simola, 2005). However, the similarities that *do* exist between the Finnish education system and the system in Quebec in the area of mathematics should serve to generate as much interest among Canadian educators in the latter as in the former, given that the social and political realities in Quebec are more easily comparable to those of the other Canadian provinces.

### **The Role of Teacher Education**

In order to be effective, a problem-solving approach to the teaching of mathematics requires good teachers as much as it requires good problems. As Richards (2014b) suggests, the *quality* of instruction, as opposed to the quantity, is likely a major factor contributing to students' success in mathematics. Recent research supports the claim: For instance, in a study based on a nationally representative sample of 194 German tenth-grade mathematics classes, Mareike and colleagues (2003) found that 14% of the variance in mathematics achievement (and 33% of the variance in enjoyment) could be attributed to instructional practice. Closer to home, in a study that focused on eighth-grade mathematics classrooms in the United States, Wenglinsky (2002) found that the effects of classroom practices were comparable in size to those of student background (such as socio-economic status), suggesting that "teachers can contribute as much to student learning as the students themselves" (p. 1). Similarly, in a summary of research on the topic, Darling-Hammond (2000) noted that when aggregated at the state level, teacher quality variables appear to be more strongly related to student achievement than variables such as class sizes, overall spending levels, and teacher salaries.

Therefore, as Li and Even (2011) suggest, understanding and implementing effective practices used for developing teachers' expertise in mathematics instruction should be of primary importance to those who care about improving mathematics classroom

instruction. This warrants an examination of the teacher education programs offered in Quebec. To start with, individuals in Quebec who wish to teach in either a primary or secondary school apply directly to a Bachelor of Education program, which gives universities four years to educate future teachers (Bednarz, 2012). As a result, students in the Bachelor of Secondary Mathematics Education program at the University of Montreal, for instance, take more than 50 credit units of mathematics courses, including courses related to the teaching of mathematics; at the University of Quebec at Montréal (UQAM), students in the corresponding program will take more than 60. It is worth noting that some studies (e.g., Begle, 1979; Ferguson & Womack, 1993) have shown a positive correlation between the amount of coursework completed and teacher performance. However, an increase in the overall number of credit hours taken does not necessarily correlate to an increase in teacher effectiveness. As many of these studies reveal, it is the *education* coursework that makes the difference. For instance, Begle (1979) found that the number of credits a teacher had obtained in mathematics methods courses was a stronger correlate of student performance than was the number of credits in mathematics courses or other indicators of preparation. Similarly, Ferguson and Womack (1993) found that the amount of education coursework completed by teachers explained more than four times the variance in teacher performance (16.5%) than did measures of content knowledge (less than 4%).

Indeed, education coursework constitutes a major part of most Bachelor of Education programs in Quebec, as in the programs offered at other institutions across Canada. However, what truly differentiates the mathematics education programs offered at many universities in Quebec from most others offered at institutions across Canada is that both the mathematics methodology *and* the mathematics content courses in the programs were developed and are taught by mathematics teacher educators (known as *didacticians* in the Province of Quebec), who also supervise pre-service teachers during their internships (Bednarz & Proulx, 2005; “Enseignement des mathématiques,” n.d.). According to Jaworski and Huang (2014), didacticians of mathematics are “mathematics (teacher–) educators who work with practicing teachers to promote developments in teaching and learning mathematics: the term includes university faculty, teaching researchers, curriculum development coordinators, master teachers, mathematics coaches, and so on” (p. 173). Similarly, the didactics of mathematics is concerned with theoretical and practical issues related to mathematics curricula and teaching, as well as their relationships with learning

(Kieran, Krainer, & Shaughnessy, 2013). It parallels Shulman's idea of "pedagogical content knowledge," sometimes referred to as "mathematics for teaching" (NMAP, 2008), which he describes as "the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman 1987, p. 8). This type of knowledge is considered to be distinct from subject matter/content knowledge and pedagogical knowledge (Liljedahl et al., 2009; Shulman, 1987). For instance, in the context of mathematics education, content knowledge encompasses mathematical concepts, the use of mathematical techniques, mathematical reasoning, and proof. Pedagogical knowledge, in contrast to pedagogical content knowledge or didactics, is independent of subject and deals with general principles of education (including theories of learning; classroom management; and the sociological, psychological, and ethical aspects of education) as well as its functions (Liljedahl et al., 2009). Didactics, then, captures the distinction between knowing something oneself and being able to teach it to others (Liljedahl et al., 2009).

Learning to teach mathematics requires a balance between these three strands of knowledge—knowledge of mathematics, knowledge of teaching mathematics, and knowledge of psychology and pedagogy (Novotná, 2009). Unfortunately, too many teacher education programs only focus on the first and the last strands, despite calls by many researchers to give teachers ample opportunities to learn pedagogical content knowledge (or didactics, as it is commonly referred to in French-speaking institutions) (Liljedahl et al., 2009; Neubrand, Seago, Agudelo-Valderrama, DeBlois, & Leikin, 2009; NMAP, 2008; Novotná, 2009). The potential benefits for students are not hypothetical; for instance, a study conducted in Germany showed that students' results in a longitudinal component of PISA were positively affected by the pedagogical content knowledge of their teachers (Brunner et al. as cited in Neubrand et al., 2009). An earlier German study conducted by Mareike et al. (2003) showed similar results, with the additional finding that pedagogical content knowledge affects not only students' achievement, but also their motivation—specifically, their enjoyment of the subject.

However, mathematics education programs instead frequently require prospective teachers to obtain mathematical knowledge that is of a more academic nature (i.e., akin to "pure" mathematics), often in courses taught to a wide spectrum of mathematics, engineering, and science students—in other words, in a form that is not obviously relevant

to secondary mathematics (da Ponte et al., 2009; Liljedahl et al., 2009). Because of this, teachers may find that they often lack the experience of how to convert formal mathematics into school mathematical activities—presumably because they should intuitively know how to do this through their experience with academic mathematics (Gellert et al., 2009). However, as Gellert and colleagues (2009) argue, school mathematics can be regarded as an *autonomous* body of knowledge and not just a simplistic form of academic mathematics. For instance, the authors point out that while academic mathematics tends to define mathematical concepts symbolically and avoids redundant formulation, school mathematical knowledge comprises many diverse representations of a particular concept, as well as the translations between them. In other words, while the research mathematician focuses on elegance and compression, the teacher focuses on *unpacking* mathematical ideas in order to make them more accessible to students (Neubrand et al., 2009).

Teacher educators in Quebec have recognized this important distinction, drawing from didactics research that has been conducted in the province since the 1970s. For instance, the required mathematics content courses taken by prospective primary school teachers in Quebec are taught by didacticians at the majority of the universities offering a primary education program (Bednarz, 2012). Having been developed specifically for future primary education teachers, these courses aim to bridge the gap between “academic” mathematics and “school” mathematics by focusing not only on the conceptual aspects of the content but also on its historical and epistemological aspects (Bednarz, 2012). Mathematical activity, which is centred on problem solving, is another important focus of the various programs (Bednarz, 2012).

Greater differences exist between the programs developed for future secondary school teachers. Future secondary mathematics teachers in Quebec will take between seven and 15 mathematics content courses (12 on average), and in most cases, these are taught by professors in the mathematics department (i.e., by “mathematicians”), as is the case in many other post-secondary institutions in Canada (Bednarz, 2012). (According to Bednarz [2012], however, new courses are starting to appear in each of these universities that seek to make connections between the courses the future teachers take and the mathematics that they will be teaching.) Among the many teacher education programs for secondary mathematics teachers in the province, however, one stands out as a trailblazer in its field. At UQAM, only *one* of the required mathematics content courses is common to secondary education students and students pursuing another degree; the other required

courses are taught by educators in the didactics department, developed specifically for future secondary mathematics teachers (Bednarz, 2012).

As such, the program at UQAM is not a simple juxtaposition of mathematics and education courses. The teacher educators (didacticians) in the program form a unified team whose aim is to develop students' proficiency in the three strands of knowledge described above (content, pedagogy, and content-specific pedagogy/didactics), as well as to bridge the gap between theory and practice (Bednarz, 2001). As Bednarz (2001) explains, pre-service teachers often spend time working on problems that would be presented to students at the secondary level. However, as Bednarz notes, they are often asked to go beyond simply solving such problems—very often, they are also required to present at least two different ways of solving them (e.g., solving algebraic problems both algebraically *and* arithmetically). As Bednarz explains, this builds the future educators' capacity to anticipate different ways of approaching problems and the difficulties that their students may encounter, and reflects the program's focus on *school* mathematics, rather than on academic mathematics. Involving pre-service teachers in such activities of translation between different representations has shown to be useful for both future primary school teachers and future secondary-school teachers (Gellert et al., 2009).

The involvement of didacticians in most aspects of the program also creates coherence and coordination between the different courses and between the courses and practicum. As a result, many professional competencies are studied in *several* courses during the four-year program (Bednarz, 2001). In particular, as the university's website explains, all didactics courses were created with the subsequent internships in mind. This reduces the danger of pre-service teachers' knowledge becoming "compartmentalized," where mathematical content knowledge, didactical knowledge, and experiences that form practicum have no or only weak connections. As Bergsten and colleagues (2009) assert, the integration of practicum with other course components is especially critical in developing a unified organization of an educational knowledge in mathematics that merges the divide between content and didactical knowledge.

The strong influence of didactics research that has been conducted in Quebec since the 1970s is also evident in the program's emphasis on examining learning situations in real contexts. For instance, pre-service teachers often work with samples of actual students' work collected by didacticians over the past four decades (including examples of student reasoning, common errors, assessment of students' work by previous teachers,

videos of lessons, etc.; Bednarz, 2001; Bednarz & Proulx, 2005). Video, in particular, has been found to promote elaborated reflection on teaching, allowing prospective teachers to focus on students' thinking and on instructional methods that make student thinking visible (Santagata & Guarino, 2011). The idea, according to Bednarz and Proulx (2005), is for future teachers to develop the analytical skills necessary to understand students' reasoning, underlying conceptions, and difficulties in mathematics, and to understand and analyze the outcomes of learning situations. Bednarz (2001) also explains that, in accordance with the emphasis on real learning situations, practising teachers are involved in aspects of the program including and *beyond* internship. As an example, students in their first didactics course at UQAM are required to prepare and present a lesson to an audience of peers, professors, *and* practising mathematics teachers. As such, Bednarz notes, students benefit from the realistic perspective of teachers who can comment on the practical aspects of the lesson, including the time and resources available, the problems that may arise, as well as the potential for student engagement and interest. The involvement of practising teachers in the program reflects a more general growing interest in increasing collaboration between teachers and academic researchers, which is seen as a means of developing knowledge about mathematics teaching and learning (Potari, Sakonidis, Chatzigoula, & Manaridis, 2010).

As explained above, the program at UQAM is not wholly representative of *all* secondary mathematics education programs in Quebec, though the university does boast the biggest Bachelor of Education program in the province in terms of student enrolment (Nadine Bednarz, personal communication, September 23, 2015). As such, we do not seek to suggest that the characteristics of certain teacher education programs in the province can fully explain Quebec students' international success in the domain of mathematics. However, we do feel that teacher educators across the country can benefit from examining the various aspects of the program at UQAM (and, indeed, of the programs available at other universities in Quebec that are almost certainly influenced by the program at UQAM and by education research conducted at the university since the 1970s) in order to improve teacher education in their respective provinces. Although such an analysis has already been undertaken by several researchers in Quebec (e.g., Nadine Bednarz, Jérôme Proulx), the topic seems as yet to have largely escaped the interest of educational researchers in other parts of the country (perhaps in part because much of the research that has been published on the subject is only available in French).

## The Role of Mathematics Teachers' Associations

It is clear that, in general, teacher educators in Quebec aim to prepare prospective mathematics teachers to the fullest extent possible for their future careers. After they have obtained their diplomas, however, mathematics educators in Quebec do not find themselves without support or opportunities for further professional development. Besides coordinating the mathematical contests described above, associations like the Association des promoteurs de l'avancement de la mathématique à l'élémentaire (Association of Advocates for the Advancement of Mathematics in Elementary Schools; APAME) have supported the teaching and learning of mathematics in many other ways: for instance, APAME regularly organized conferences during which educators could discuss issues related to the teaching of mathematics at the elementary level; it also published the journal *Instantanés mathématiques* (Mathematical Snapshots), which offered suggestions for projects and activities, reports of in-class experiences, and other diverse articles relevant to the teaching of elementary-level mathematics (Dionne, 2007). (Note: Similar associations in other Canadian provinces fall under varied headings, including organizations, societies, and groups [e.g., The Saskatchewan Mathematics Teachers' Society].) According to Dionne, for more than 40 years, APAME was an essential part of the "life" of mathematics in Quebec and of its evolution. Unfortunately, APAME was forced to cease its operations in the early 2000s due to financing and other issues (Dionne, 2007, 2002).

However, a considerably wide range of other associations that support both primary and secondary teachers of mathematics in Quebec are not only still operating but are thriving. These include the Groupe des responsables en mathématique au secondaire (GRMS, established in 1973), which supports teachers of mathematics at the high school level, the Association Mathématique du Québec (AMQ, established in 1958), which brings together research mathematicians, as well as all those who are interested in the teaching, development, and popularization of mathematics, and the Groupe de didactique des mathématiques du Québec (GDM, established in 1970), which is devoted to the professional development of mathematics teachers in the province. The GRMS is a particularly strong presence in the Quebec mathematics education scene—what started as a committee in 1969, whose role was to specify the needs related to the teaching of mathematics at the secondary level and to serve as an interlocutor between teachers and the Quebec Ministry of Education, has evolved into an active professional association that

supports teachers of mathematics in a multitude of ways. Among its many activities, the GRMS has created a “mathematical briefcase” containing mathematical strategy games and recreational mathematical problems, which was distributed to schools, shopping centres, and public science exhibitions in the 1970s with the aim to promote “scientific leisure activities”; since the 1980s, it has been publishing the biannual journal *Envol*, which includes articles related to the teaching of mathematics as well as mathematical problems and information for members about upcoming conferences and events; and, since 1988, it has been organizing the annual mathematical contest Opti-Math for secondary students (described in a previous section). In addition, the GRMS encourages excellence in teaching by offering annual scholarships to teachers of mathematics who display “enthusiasm, leadership, innovation, quality of teaching, and/or influence,” as well as prizes to promising students entering the profession who have graduated from an affiliated university. According to the GRMS website, supporting beginning *and* practising teachers—which has taken the form of not only offering scholarships and professional development opportunities but also of involvement in research related to teacher education—has always been a primary focus of the organization.

While the reach of the GRMS extends to all those interested in secondary-level mathematics, the goals of the AMQ are wider in scope. On its website, the AMQ describes its ambitions as follows: to generate public interest in mathematics through a range of activities and publications; to support progress related to the teaching of mathematics through collaborations with the Ministry of Education, educational institutions, and publishers; and to support educators in their work by making available a diverse array of services (including conferences for teachers and mathematical contests and camps for students). As such, the AMQ functions partly as a parent organization for the mathematical associations in Quebec, with affiliates including the GDM, the APAME, and the GRMS (the latter was, in fact, originally an offshoot of the AMQ).

What emerges from this brief study of the rich network of mathematics and mathematics teacher associations in Quebec is a clear commitment to the professional development of teachers at both the primary and secondary levels that is based on current educational research. It is evident that, as Dionne (2002) contends, the teaching of mathematics has long been and continues to be a major concern and preoccupation in Quebec. Dionne (2002) also suggests that the efforts of these associations have contributed to students’ successes in recent years, which is certainly plausible, given that much

research has shown a positive effect of professional development on teachers' instruction (e.g., Desimone, Porter, Garet, Suk Yoon, & Birman, 2002; Suk Yoon, Duncan, Wen-Yun Lee, Scarloss, & Shapley, 2007). This view is shared by Quebec's Conseil supérieur de l'éducation (CSE), which points to the continual professional development of teachers as a critical factor in students' performance (CSE, 2014). Of course, this is not to suggest that mathematics educators in the other Canadian provinces are not concerned with professional development, nor that they lack support—mathematics teachers' associations do exist in most of the other provinces, and these associations all offer some combination of professional development, networking opportunities, monetary support (in the form of scholarships), and information (e.g., journals or newsletters) to mathematics teachers in their respective provinces. What sets the associations in Quebec apart are their levels of activity and relatively extensive involvement in the life of mathematics education in the province (see discussion above). Moreover, unlike in the other provinces, there are at least *three* such associations in the Quebec, which suggests, at the very least, more opportunities for professional development and networking for mathematics educators in the province.

## Discussion

Based on the evidence presented above, we contend that students' success in mathematics cannot be boiled down to a simple formula. Nonetheless, the impressive performance of Quebec students—on both national and international scales—can be attributed, at least in part, to several aspects of the educational system in Quebec. As we have detailed, these aspects include diverse opportunities for students to participate in recreational mathematics activities; an emphasis on problem solving in the classroom; intensive teacher education programs that focus not only on content knowledge but also on *pedagogical* content knowledge in the area of mathematics; and an active network of mathematics and mathematics teacher associations that support research and professional development in education through conferences, publications, and other activities. Interestingly, although many journalists attribute Quebec's success in this domain to an emphasis on “traditional” teaching methods and ensuring that students know the “basic facts” of mathematics, none

of the literature reviewed for this report suggests that this is the case. Indeed, it rather indicates that the pendulum generally swings toward the “reform” side in Quebec.

However, the Math Wars, which have pitted traditional and reform mathematics against each other for many long years, seem to be preventing the public and the government from considering many other important factors that can improve the teaching and learning of mathematics across the country. To start with, high-quality teacher education should be the focus of any program aimed to improve students’ achievement in mathematics, and studying the models currently employed at universities such as UQAM seems to be a good place to start. At the school level, *quality over quantity*, as the saying goes, should guide policy makers in the process of improving mathematics education. And yet, one of the latest educational policy changes in Saskatchewan was an increase in instructional time to 950 hours per year, a change which required some school boards to add as many as 50 hours to the school year (McMahon, 2014). The increase was made in an attempt to boost student achievement, despite the fact that many studies have shown that an increase in instructional time in and of itself does not improve student performance in mathematics. According to the 2012 PISA results, for instance, the average time devoted to regular mathematics instruction ranged from 257 minutes per week in Newfoundland to 364 in Alberta among the Canadian provinces; however, the correlation between average instruction time and mathematics performance was found to be statistically negligible (Richards, 2014b).

Moreover, as Sir David Spiegelhalter (Winton Professor for the Public Understanding of Risk at the University of Cambridge) and other researchers stress, there is plenty of reason to take the results of international assessments—including PISA—with a grain of salt (see, e.g., Spiegelhalter, 2013). As Brown and Clarke (2013) point out, governments have been jockeying for a better position in the league tables of such assessments, with good performance in these tables sometimes being interpreted as being indicative of wider economic competitiveness. However, such comparisons can transform the content of what they compare. In particular, some interpreters of these assessments cast students as “passive, nameless metaphors of national economies, whose performance in school will predict the future relations among nations” (Thorsten, as cited in Brown & Clarke, 2013, p. 460). More and more, global politics motivate policy makers to apply national security responses to education when considering the results of such international assessments (recall John Manley’s view that the latest PISA results constitute a “national

emergency”; Brown & Clarke, 2013). Unfortunately, there is little questioning in Canadian political circles about what these assessments really measure or how comparable they are across national, or even provincial boundaries. Moreover, both policy makers and the media are prone to drawing quick conclusions based on assessment results, especially if a ranking is lower than the previous cycle; sometimes, as the decision to increase in instructional time in Saskatchewan shows, this leads to knee-jerk policy changes that are not supported by educational research. Such actions are typically politically motivated and short-minded—for example, instead of focusing on long-term investment for a better education, policy makers often opt for “quick fixes” to improve test scores that can be explained in “electorate-friendly” terms (Brown & Clarke, 2013; Dossey & Wu, 2013). What is often absent from discussions in the media related to such assessment results, however, are the margins of error that are associated with any reported measure. As Dossey and Wu (2013) explain, although those who conduct international studies take great pains to articulate the level of confidence surrounding performance measures, these margins of error are often ignored—even if the change in a ranking is likely simply the result of random fluctuation due to the sampling of students.

We recognize the potential usefulness in using international assessments to evaluate the quality of mathematics education in various regions. However, it seems that more often than not, the results serve only to obfuscate issues surrounding the teaching and learning of mathematics among the general public and to entrench deeply-held beliefs about the merits of “traditional” approaches, which focus on rote practise and memorizing rules, algorithms, facts, and relationships, as opposed to emphasizing conceptual understanding and applied problem solving. We propose that, instead, they are taken as potentially useful clues, or suggestions, that stimulate critical reflection and lead those interested in the betterment of mathematics education toward collaboration—rather than competition—with educational systems in regions like Quebec that are arguably on the right track. We must also accept that change will be slow and that educational reform is a complex, multifaceted process that cannot be accomplished with a handful of policy changes based on tradition, gut feelings, and “common sense.” Rather, it is informed decisions based on evidence and research that may resolve the so-called Math Wars, which continue to cause casualties: our children, who may not receive the kind of robust mathematics education that they deserve (Schoenfeld, 2004). Change has been a long time coming, and in this situation, it seems that *plus ça change...plus ça paie*.

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