Attitudes of Secondary Students Concerning the Nature of Mathematics, Mathematics Teaching, and Learning Related to Achievement

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The relationships between certain student background variables, aptitude, classroom variables as measured by the learning environment inventory, achievement, and the seven attitude variables developed by the International Association for the Evaluation of Educational Achievement are examined. The sample consisted of 16 general and 32 advanced level Grade 9 and 10 mathematics classes in the Province of Ontario. The results suggest that in regular classes higher achievement is associated with teacher-centred methods of instruction and the attitude that mathematics is a closed, fixed system, whereas in advanced level classes these relationships are reversed. Generally correlations between attitudes and the other variables in this study are low. The validity of some of the IEA scales is questioned.

Cet article examine les rapports existant entre certaines variables liées aux antécédents des élèves, leurs aptitudes, des variables telles que mesurées par les composantes de l'environment dans lequel se fait l'apprentissage, la variable réussite et les sept variables d'attitude élaborées par l'*International Association for the Evaluation of Education Achievement*. L'échantillon comprenait 16 classes ordinaires et 32 classes enrichies de mathématiques de 9e et 10e années en Ontario. Les résultats font voir que, dans les classes ordinaires, les notes élevées vont de pair avec une pédagogie centrée sur l'enseignant et la conception selon laquelle les mathématiques constituent un système fermé et fixe tandis que, dans les classes enrichies, ces rapports sont inversés. Les corrélations entre les attitudes et les autres variables de l'étude sont en règle générale faibles. L'article met en doute la validité de certaines échelles de l'IEA.

Although pupils' attitudes towards mathematics are thought to be important factors in learning the subject, major reviews of research on the teaching of mathematics have often ignored the topic entirely (Henderson, 1963; Romberg, 1969). Early studies in this field typically were limited to an assessment of students' interests in or liking for the subject, the class, or the teacher (Dutton, 1956, 1968).

Current research has identified many facets of student attitudes towards mathematics and has examined them in a variety of contexts (Aiken, 1970, 1972, 1976). A recent review by Aiken (1976) cites studies which investigated one or more of the following: perceptions of teaching, perceptions of the teacher, anxiety (mathophobia), value of mathematics in society, enjoyment, and interest in mathematics. These facets of attitudes towards mathematics are studied in relation to student development, sex of student, student self-concept, achievement, personality attributes, and certain instructional and curricular factors. Aiken's review concludes with the admonition that many personal and situational characteristics influence student attitudes and that any investigation should take into account several such possible sources of variability (Aiken, 1976).

This study examined the attitudes and beliefs of advanced and general level secondary mathematics students in Ontario. In keeping with Aiken's suggestion, the attitudes were studied in relation to student characteristics, classroom environment, and student achievement. The specific attitudes under investigation were: (a) students' views of the methods employed by teachers of mathematics, (b) students' beliefs about the nature of mathematics, and (c) attitudes about the importance and utility of mathematics. The scales used to measure these attitudes were developed by the International Association for the Evaluation of Educational Achievement (IEA), which conducted a 12-nation cross-national study of mathematics achievement in 1963-64 (Husén, 1967).

A secondary purpose of this study was to re-examine some of the findings of the IEA project which have not been satisfactorily explained. The data from the IEA study indicate positive correlations between students' view that teachers use "discovery techniques" and the mathematics achievement of 13-year-olds, but small and negative correlations for 18-year-olds (Husén, 1967, Vol. 2, pp. 145-149). Husén suggests that the latter result may derive from either conservative methods of teaching or the nature of the examination system (Husén, 1969, Vol. 2, p. 299). Neale (1969) analyzed the data from the American students who participated in the IEA study and found no such relationships between achievement and the view that teachers use "discovery techniques." In Ontario at the time of this study, there were no important centrally imposed examinations for secondary school students; therefore Husén's explanations were not applicable.

A surprising result of the IEA study calls for a serious re-examination of the nature of mathematics as viewed by secondary students. Generally there was a negative correlation between achievement and the belief that mathematics is an open system. The association of scores was stronger for older students. Students who were most interested in mathematics tended to view it as a fixed, formal, closed system. Interest was measured by responses to six items through which respondents indicated their liking of the subject and their desire to take additional courses in mathematics (Husén, 1967, Vol. 1, p. 212). This is the only well-known scale which measures the belief. The above findings are contradictory to the assumptions of current mathematics curriculum theories and should be examined further.

DESIGN OF THE STUDY

The unit of analysis in this study is the classroom, and all scores are mean classroom scores. Forty-eight classrooms from 12 secondary schools from four boards of education in eastern Ontario constituted the sample. To fulfill certain requirements of the major project (O'Reilly & Garland, 1974) from which the data for this study were drawn, a combination of random and judgmental sampling was employed.

The 48 classes were equally divided between the Grades 9 and 10 levels (or equivalents). Two-thirds of the classes at each grade level were advanced academically-oriented and the remainder were general or terminal. This particular distribution was not planned; since the majority of the classes are advanced, the descriptive data (means) should be interpreted cautiously. A total of 1,100 students participated.

Data collection. Data were collected from students with the following instruments: (a) Stanford Achievement Test (SAT): Mathematics Form W, (b) Parents' Education Inventory, (c) Learning Environment Inventory (LEI), (d) School and College Ability (SCAT) Test, Series II, and (e) School Mathematics Inventory.

Questionnaires and tests were administered in two sittings with each of the 48 classes. One sitting involved the administration of the Stanford Achievement Test: Mathematics, Part A, and the appropriate forms of the SCAT, Series II. The Stanford Achievement Test was administered to two-thirds of the class while the SCAT was administered to the remaining one-third. In the other sitting, the Learning Environment Inventory and the Parents' Education Inventory were administered to one half of the class while the School Mathematics Inventory was administered to the remainder of the class. Males and females were proportionally represented in each segment of data collection.

The order of administering sets of instruments in the two settings was alternated to eliminate order effects.

The Learning Environment Inventory was developed by Anderson and Walberg (1974) for the evaluation of the Harvard Project Physics. The instrument consists of 105 items describing typical school classes. Each student expresses his agreement or disagreement on a four point scale. The items are grouped into 15 scales.

The intra-class correlations for class groups ranged from .31 to .92 for each scale. The LEI was successfully used in a variety of experimental and correlational studies to relate classroom climate to teaching methods and to achievement (Anderson & Walberg, 1974).

The first IEA attitude variable defines the teaching methods employed. *Views about mathematics teaching* ranges from a view of the teaching-learning situation which stimulates students by means of an inquiry approach to a view that teachers employ mechanistic, formalistic modes with great emphasis on rote memorization. The possible range of scores is from 0 to 22 for this 11-item scale. High scores indicate an inquiry approach is perceived.

The second descriptive variable defines the climate of the school and school learning. *Views about school learning* describes situations which vary from a teacher-centred, authoritarian-based program to one which promotes inquiry methods, pupil activity, and discovery. The possible range of scores is from 0 to 22.

The IEA analysis assigned to each student the mean score achieved by his class on these first two scales and in their analysis referred to the combined score as an index of the use of inquiry-centred methods of teaching (Husén, 1967, Vol. 2, pp. 148-157).

The underlying continuum for *Attitudes toward mathematics as a process* ranges from a view that mathematics is a fixed, formal system governed by rigid and unchanging rules which a student had to master, to a view that mathematics is a subject that is still in a process of development. There are eight items and scores range from 0 to 16.

The underlying continuum for *Attitudes about the difficulties of learning mathematics* ranges from a view that only few can learn it to a view that mathematics is not difficult. This is a seven-item scale. High scores indicate agreement with the former view.

The underlying continuum for the scale measuring *Attitudes toward the place of mathematics in society* ranges from a view that mathematics is neither essential nor useful to a view that mathematical knowledge is important not only in terms of obtaining a good job but also in terms of national development. The possible range for this eight-item scale is from 0 to 16.

The scale measuring *Attitudes toward school and school learning* suggests a range from dislike for schooling and a desire to leave it to a genuine liking for school work. This scale comprises 11 items.

The scale measuring *Attitudes toward man and his environment* assess the student's view of the relationship between man and his environment. According to Husén, low scores suggest the belief that man is at the mercy of his social and physical environment. An examination of the items suggests that the scale also measures the student's acceptance of the rationalist assumptions that education, medicine, engineering, and scientific research will solve most of the world's problems. The range of scores is from 0 to 18.

The reliability of each scale was established in pilot studies for each nation for each of two samples: 13-year olds and pre-university year students. Guttman scale analysis coefficients of reproducibility range from .77 to .92. For the final study, most scales are estimated to have coefficients ranging from .85 to .90 (Husén, 1967, Vol. 1, pp. 114-119).

FINDINGS AND DISCUSSION

The average scores for this sample of students, most of whom fall within the 14-16 age group, do not appear to differ in any appreciable manner from the means obtained by the IEA 10 years earlier. Where differences exist, they indicate that Ontario students feel that mathematics is easier and more important than did IEA samples. Ontario students also seem to enjoy school more and to have more faith in the efficacy of science (Table 1). The fact that the Ontario sample is not randomly drawn and is composed mainly of advanced level classes probably influences these results.

TABLE 1

Means of Attitude Scales for Ontario Subjects, United States, and 10 Nations Total

Variables[a]	Ontario Subjects	United States[b] 1a	United States[b] 3a	10 Nations[b] 1a	10 Nations[b] 3a
1. Mathematics teaching (11)[a]	12.5	23.8	24.7	23.8	23.8
2. School learning (11)	11.8				
3. Mathematics as process (8)	7.5	8.0	7.4	7.7	6.6
4. Difficulty (7)	10.9	9.4	8.5	9.1	8.2
5. Importance (8)	9.6	8.6	7.1	8.8	8.1
6. Attitudes toward school (11)	11.6	8.4	8.0	9.3	9.2
7. Man and environment (9)	10.6	7.9	7.4	8.6	8.0
n	535	6,231	1,568	27,228	9,007

[a] The numbers in brackets following the variable names indicate the number of items used to measure each variable. The range for each variable is 0 to twice the number of items.

[b] Populations 1a are 13-year-olds; populations 3a are mathematics students in final secondary year. Data for U.S. and 10 nations are from Husén (1967, Vol. II, ch. 4). In the IEA study, scales 1 and 2 were collapsed into one scale for purposes of analysis.

Attitudes and Input Variables. Two input variables are considered here: scholastic aptitude as measured by the School and College Ability Test (SCAT), Series II, Forms 2A and 3A, and level of education attained by the parents of the students.

Aptitude scores are associated with one descriptive variable, the view that school and school learning are oriented toward student activity rather than toward teacher-domination. Correlations between this view and mathematical aptitude is .50 ($p < .01$) and between this view and verbal aptitude is .32 ($p < .05$). Similarly the level of parents' education is associated with this view ($r = .30$, $p < .05$). These results suggest that teachers of classes composed of brighter students permit more opportunities for students to initiate activities and to participate in discussions. Alternatively, such students may simply perceive more of these opportunities than do students with lower aptitude scores.

TABLE 2

Correlations Between Class Mean Scores for Input Variables and Attitudes to Mathematics and Mathematics Learning

Attitudes	*Input Variables* SCAT Total	*Parents' Education*
1. Mathematics teaching	.22	.00
2. School learning	.45*	.30*
3. Mathematics as process	.21	—.12
4. Difficulty	.16	—.10
5. Importance	.31*	.09
6. Attitudes toward school	.13	—.11
7. Man and environment	.10	—.14

NOTE: $n = 48$
* $p < .05$

Generally, these input variables are not related to the other attitudinal variables except that in classes composed of brighter students, mathematics is thought to be important. Thus, student background factors are associated with student views of school learning; since the other attitude scores are not related to background factors of aptitude and home environment, it is suggested that they are learned at school, albeit as influenced by other idiosyncratic variables.

Attitudes and Classroom Climate. Classroom climate as measured by the Learning Environment Inventory (LEI) is highly correlated to the two descriptive measures developed by the IEA. The multiple R between the LEI scales and each of the descriptive variables is .75 (Table 3). The most significant relationships are between the LEI scales and the view about school and school learning scale. Student activity oriented classes tend to be more cohesive, more organized, more democratic, and less formal than teacher-dominated classrooms. In the inquiry-centred classes pupils possess a more favorable self-concept, they like their classes, they perceive less conflict in their classes, and they perceive their classrooms as adequate for their needs. The same LEI variables are associated in the same manner with achievement (O'Reilly, 1975).

None of the other IEA scales are significantly correlated with the classroom environment scales of the LEI. Thus the two IEA descriptive measures are perhaps less measures of student attitudes and more indices of classroom climate and teaching style. Classroom climate is generated by the interaction between pupil and teacher variables.

TABLE 3

Correlations Between Learning Environment Inventory Scales and IEA Descriptive Scales

Environment Scales	*Descriptive Scales*	
	Views About Math Teaching	*Views About School Learning*
1. Cohesiveness	—.02	.36*
2. Diversity	.16	—.10
3. Formality	.15	—.37*
4. Speed	—.24	—.08
5. Environment	.24	.37*
6. Friction	—.42*	—.30*
7. Goal Direction	.07	.14
8. Favoritism	—.07	—.33*
9. Cliqueness	.09	—.17
10. Satisfaction	.22	.35*
11. Disorganization	—.25	—.33*
12. Difficulty	—.10	.17
13. Apathy	—.16	—.40*
14. Democratic	.09	.48*
15. Competitiveness	.12	.22
R	.75*	.75*

* $p<.05$

Methods of instruction and achievement. The hypotheses to be tested are that inquiry methods of teaching are associated with higher levels of achievement in mathematics and that classes oriented toward student activity achieve higher mathematics scores than teacher-dominated classes. Although many educators take these hypotheses for granted, large scale field studies supporting these claims are rare.

The results of the IEA study did not support the hypotheses. Treating the two descriptive scales as one summative scale, the IEA researchers reported small positive correlations between achievement and inquiry-methods of teaching for younger students and small but negative correlations between these two variables for older students. Although some countries produced correlations larger than .08, the level required for statistical significance, most correlations were not significant. The Ontario data show that in classes in which students perceive that teachers employed student activity oriented methods there was higher achievement than in classes in which students perceived that teacher-dominated methods were employed. There is a significant positive correlation between class means of achievement and the *Views about school learning* scale ($r = .56$, see Table 4).

TABLE 4

Correlations of Mathematics Achievement with Student Views of Mathematics Teaching and School Learning and Student Beliefs about Mathematics as an Open Process for General and Advanced Level Grades 9 and 10 Classes

	General	*Advanced*	*Total*
	$n = 16$	$n = 32$	$n = 48$
Mathematics teaching	—.33	.12	.20
School learning	.09	.60**	.56**
Mathematics as process	—.01	.47*	.26

* $p < .05$
** $p < .01$

An argument can be made that teachers make a priori assumptions about their classes and allow students more or less initiative depending on the student's social and ability characteristics. Note that the descriptive scale which correlates positively with achievement across all sub-groups, *Views about school learning*, is also related to level of parents' education and to pupil aptitude. Further, although the means of the two descriptive variables are similar for both Grades 9 and 10 of the Ontario sample, the means for the advanced course classes are higher than for the general course classes. Again, the alternative hypothesis to consider is that the more capable students perceive that classes are more student activity oriented than do their less capable peers.

Since the correlations between the descriptive scale *Views about mathematics teaching* and achievement tend to be non-significant, any interpretations must be viewed as highly speculative. It does appear that for classes of lesser ability (Ontario general level program students), mechanistic, rote methods of teaching are associated with higher achievement. One possible reason for this, in the case of less competent students, may be that they have not developed the required skills to be successful with inquiry methods.

The fact that achievement of pupils is not related to the views about teaching in both the Ontario and IEA studies may be due to a possible lack of validity of the scale itself. Husén (1967, Vol. 2, p. 152) raises this possibility. The strong correlations between the two descriptive scales and the LEI scales (Table 3) suggest that they are more indicators of classroom climate than measures of teaching methods. The scale *Views about school learning* in particular appears to measure whether or not the teacher insists on teacher and text centred learning instead of directly assessing whether or not inquiry methods are employed.

Attitudes and achievement. Attitudes towards a subject should influence the amount of effort expended in learning a subject, and thus influence achievement. On the basis of the IEA data it was concluded that affective outcomes of teaching are not consistently related to cognitive outcomes. In this study, the only attitude which is related to achievement is the one that mathematics is important to society ($r = .30$).

TABLE 5

Correlations Between Mathematics Achievement and Attitudes Over All Nations and for Ontario

	IEA Populations				
Variables	*1a*	*3b*	*3a*	*1b*	*Ontario*
Difficulty	.00	—.02	—.07	.00	.08
Importance	.04	.08	.09*	.13*	.30*
School learning	—.16	—.14	.06	*a*	—.01
Man and environment	—.18	—.12	*a*	*a*	.02

NOTE: Correlations for the difficulty and importance variables are from Husén (1967, Vol. 2, Table 4.6). Correlations for the remaining variables are median correlations of the 12 nations' correlations. See Husén (1967, Vol. 2, Table 1.12); *a* indicates low or erratic correlations.

* $p < .05$

The attitude that man can control his social and physical environment through the use of science and knowledge (*Man and environment*) tended to be negatively related to achievement in mathematics in the IEA sample of younger students (Husén, 1967, Vol. 2, p. 38) but not in the Ontario study where the correlation was .02. Generally these results are similar to those reported in the studies surveyed by Aiken (1976).

Achievement and belief about the nature of mathematics. The variable *Mathematics as process* is somewhat distinct from the other attitude scales employed by the IEA as it refers to students' perceptions or beliefs about the nature of mathematics. This variable is singled out, for one main purpose of the new mathematics movement was to alter the image students have of the structure and process of mathematics. These programs seek to give students a truer image of mathematcs.

The correlation for the class means between this belief and achievement for the 32 advanced classes of students who may be preparing for college-level study of mathematics is significant ($r = .47$, see Table 4). These students would have been most consistently exposed to new mathematics curricula. The correlation for the general level students is —.01. One possible hypothesis is that the general course students, whose achievement scores are much lower than those of the advanced students, have had teachers who adopted more mechanistic methods of teaching and stressed "fundamentals" rather than permitting student initiative to explore ways in which mathematics could be considered an open system. In these classes, the openness of mathematics could be made an irrelevant issue by the nature of the teacher's style. Thus no significant correlation between this belief and achievement is expected and none was found. Another possible explanation is that advanced level students and more capable students are better able to benefit from the notion of mathematics as an open system. A final hypothesis is that less capable students are unable to conceptualize mathematics as an open system.

CONCLUSION AND SUMMARY

The results of this study contrast in specific ways the results of the IEA study. The Ontario data suggest that for Grades 9 and 10 level students, teaching methods viewed as less mechanistic are associated with higher achievement, particularly in the case of students in advanced classes. In the case of general program students, the view that teaching focuses on mechanistic methods is associated with achievement. However, even in this case, the correlation between achievement and this view ($r = -.33$) is not significant. The *Views about school learning* as student activity-centred is related to achievement, both for the total sample and the sample of 32 advanced level classes. The dissimilarity of results for the general level classes suggests that less capable students may not be given an opportunity to engage in inquiry methods or they may be less capable of profiting from such methods.

The validity of some of these scales may be questioned. For example, the IEA scale, *Attitudes about the difficulties of learning mathematics*, is not related to achievement. However, for the sample, the Walberg and Anderson LEI sub-scale "difficulty" did correlate positively with achievement ($r = .33$, $p < .05$). Similarly, the *Attitudes toward school and school learning*, which is a measure of satisfaction with school or liking for school, was not related to achievement. Yet in the same study, the LEI variable "satisfaction," which measures the same phenomena, correlates .45 with achievement for the same sample (O'Reilly, 1975). Similarly, for the same study the authors devised a scale labelled "dislike for schooling." This correlated —.41 with achievement (O'Reilly & Garland, 1974). Thus, two scales correlate in a similar manner with achievement, although an IEA attitude scale which purports to measure the same phenomena does not. Note that the IEA scales did not correlate with their respective namesakes in the LEI battery. The scales developed by Aiken (1974) to measure the attitudes of enjoyment and importance of mathematics seem to be superior to the IEA scales. The remaining attitude scale of the IEA may simply be attempting to measure concepts which are too complex to be measured by short scales. The attitude *Man and environment* appears to be an indicator of a person's orientation toward the world as Rotter attempts to do with his I/E scale (Rotter, 1954). Findings of studies based upon the IEA attitude scales appear to be most useful when the students are asked to give their views of the processes teachers use and their beliefs about the nature of mathematics. The remaining IEA attitude scales require further analysis before any definite conclusions are accepted. Generally student attitudes are related to other educational variables and should be considered as important process and outcome variables.

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